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ELECTRONICS FOR DEFENSE AND INDUSTRY*

BY

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The theme for the National Electronics Conference a year ago was "Electronics for Defense." This year your officers chose the theme "Electronics for Defense and Industry," and it is on this subject that I now address you.

I sincerely hope that your officers are prophetic in their choice of a progression of themes for succeeding conferences. I say this because this progression, if unchanged, would be as follows:

A year ago—Electronics for Defense
This year—Electronics for Defense and Industry
A future year—Electronics for Industry.

I am thinking, of course, of industry in a peacetime environment with the objective of direct benefits to mankind, for this is the ultimate objective; this is the motive that inspires.

There has just been concluded in this city a convocation commemorating the centennial of engineering in this country. Running throughout the reviews of technical accomplishments at that convocation was a common thread of thought. This common theme, no matter what field of engineering, stressed contributions towards living comforts, safety, security, and happiness. The speakers were confident that these gains would continue, that benefits would grow, that the whole of mankind would be the recipient of these advances.

We are now in the aftermath of World War II, a war that drew upon mobilized technology to an extent never before contemplated.

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We are currently in the midst of a defense program that is building more and more upon a foundation of science and technology. At the same time, we are advancing science and technology on an industry-wide front for peacetime purposes. And all the while we are trying to achieve a proper balance of effort between these objectives. One objective is preparatory defense to assure peace; the other objective is the enjoyment of that peace through the advance of man's comfort and position.

Thus, today, we are in an era of "defense and industry"—inseparable objectives in these times. Here, electronics has a key role of ever-increasing importance. Out of each dollar now spent for national defense, approximately 10 cents go for electronics, communications and related items, a substantial increase over the electronics slice in World War II.

The Defense Department is expected to earmark more than 4 billion dollars for electronic equipment in 1952. This figure will be even higher if one adds other governmental investment in electronics to support the three services. This is quite an order for an industry that in 1940 grossed well under a billion dollars. Not only in volume but in adaptability, the industry has made an impressive showing. Production of military electronic items is now at a rate seven times that existing at the outbreak of the Korean war. And all but 5 per cent of these items are of new and improved design. As an industry providing peacetime services, electronics now ranks with the leaders. In 1951, the public paid close to $5,000,000,000 for its products and services, including radio and television broadcasting.

Such activity, such expansion does not take place without adjustments. Let us scan, then, what I consider today's Number One problem in military electronics.

As new instruments and vehicles of war come into being, they are ever more complex. Electronics has grown from a simple means of contact and communication into the central nervous system of military might. More and more, electronics is assuming functions formerly performed by man's mind. This is a requirement of the increase in speed of military vehicles and carriers. Man's mind interprets vital data too slowly to keep track of many modern weapons and vehicles. The ultra speed of electronic detectors, computers, and controls is now mandatory.

Thus, military systems, which must be competitive with each other, grow more and more rapid in their operation. The reaction time becomes too fast for human operation and related factors become too numerous for rapid human comprehension. Thus electronics steps in to react, to interpret, to compute, to control, and finally, to take the place of man's senses and mind. The chains become longer, the systems more complex.

The men who operate and maintain this equipment have not changed, however. Because of electronics they are more powerful, but they, themselves, are not basically different from the man who drove the chariot or pulled the cross-bow.

Let's assume that this situation is represented by a triangle. The base of the triangle represents the man behind the anti-aircraft gun, the radar operator, or the jet pilot. This base we shall assume is unchanging in length. One leg constitutes the complexity of the system and this leg, as we know, has grown in length as the years, or even the months, have gone by. So far we have specified the base and one leg of the triangle. The base is relatively fixed—the human operator. The leg is varying in length and seemingly always becoming longer—the equipment complexity. Thus if we are to have a normal closed triangle, the other leg, which I have not so far defined, must grow in length with the complexity leg. What is this third leg of the triangle? It is the reliability factor of the equipment and system. If complexity increases, so must reliability in order to keep the triangle closed. The reliability leg must always keep up with the complexity leg if these weapons are to be of any service at all. The most ingenious circuit in the world is no good if the fighter plane that incorporates it fails on its way to meet its target.

All electronic engineers have a deep responsibility here. Let us keep in mind this simple triangle: operator, complexity, reliability. A useful result will be achieved only under the conditions of a closed triangle. Reliability, the leg that gives us the most concern, should be our major objective in military electronics.

Now let us have a brief look at today's Number One opportunity in peacetime electronics—television.

Immediately after the war, activity in extending commercial television gained great momentum. The 10,000 receivers in the public's hands at the close of the war quickly multiplied. Today, seven years later, there are more than 18,000,000 sets with an estimated audience of 60,000,000 persons. The number of stations on the air has grown from fewer than a dozen in 1946—and these offering only limited programs—to 110 serving 65 urban areas. By means of coaxial cable and radio relay the geographic spread of television programming has increased. Today we have expanding network facilities which span the continent. This has added strength and impetus to television.

At the beginning, just thirteen channels were provided for the service. These were in the very-high-frequency portion of the radio spectrum. The rapid growth of television broadcasting brought with it problems in the allocation of stations because of possible interference between them. This led to a review and there followed a "freeze" by the Federal Communications Commission on additional station assignments. The "freeze" began in September of 1948 and continued until a few months ago.

During this period, information and experience were accumulated on these interference problems. In addition, a system of slightly offsetting the frequencies of nearby stations on the same channels did much to eliminate or minimize a predominant source of interference.

The readily apparent need for more channels to achieve full coverage for television spurred fundamental studies on the ultra-high-frequency portion of the radio spectrum. A succession of propagation studies and field tests culminated in an extensive UHF set-up made by RCA-NBC at Bridgeport, Connecticut. Here tests were conducted under full-scale conditions and over a long period of time. First these were for basic studies of the system and later as a service to receiver designers.

With the practicality of UHF established and with technical considerations of VHF interference better understood and reduced, the Federal Communications Commission, in April of 1952, lifted the "freeze" on station construction. They announced the criteria for allocation of new VHF and UHF stations. By September, 1952, 60 new stations had been authorized.

This activity marks the start of the second big surge of television. The way has been opened for approximately 500 more stations in the VHF band and some 1500 stations in the UHF channels. The present skeleton of nation-wide television service can be filled out. Virtually every community in the country will be within range of television broadcasts. A total of 50,000,000 television sets in the United States within five years is not an unreasonable estimate.

Added to this promise of growth for black-and-white television, is the opportunity for television in color. Extensive testing is under way. Experience is now building up in many segments of the industry. The National Television System Committee is active on the determination of signal standards. We may move forward with a compatible system permitting an orderly progression to television broadcasting enhanced by the addition of color. By compatible, I mean a color television system that can be received in black and white on conventional television receivers without modification.

Now let us take a look at a current promise for both military and peacetime electronics. This area of research—electronics in the solid state—is just now crossing the threshold into a revolutionary future. It is my Number One promise for electronics of tomorrow.

Radio equipment of the early days made use of essentially the same materials as its older brother, the electrical industry. These were the conductors—materials permitting the ready movement of electrons when under the proper influence; insulators—materials where the electrons are bound; and magnetic materials. From almost the beginning, however, a new class of materials entered the radio scene. These were neither conductors nor insulators in the usual sense and they did not obey

Ohm's law. I refer to the loosely packed particles of the coherer and the crystal with its point contacts. These were the detectors of radio waves. While the performance of such units could be measured, the basis of the performance was little understood. Except for such specialty applications these semi-conductors were the discards of the electric and radio arts. They served well during the early radio days but passed from the stage when the electron tube emerged.

The electron tube became the lever-arm of radio and electronics. It is the keystone of all apparatus and techniques upon which the present ever-expanding industry depends.

While we have progressed far, and while we are still expanding the versatility and usefulness of electron tubes, attention again has been directed to the discard materials, the semi-conductors. As is so often the case, we find in the discard the real gem itself. But this time the approach was not through empirical experimentation but by painstaking research with understanding of each step. Also it was not a single approach but one which has taken many routes with many evidences of current and potential results.

The first broad uses of these materials came from their nonlinear and unilateral properties which were those of importance for radio-frequency detectors of the early days. Then we moved into small power applications as well. As understanding grew, we learned that conductivity could be influenced by radiant energy (photoconductivity), by electron bombardment (bombardment-induced conductivity), and by applied voltage (transistors), just as in the case for the electron tube. Here, however, we are working with the controlled action of electrons in solid materials.

Why is this important? It is important because we have a new tool, a new instrumentality. It promises to augment and to supplement the electron tube. It means new freedoms in the future in the designs of equipment. It means wider and added services and uses. It provides a new dimension.

We might well ask "Doesn't this spell obsolescence?" The answer is obviously "Yes," but it is the obsolescence which is the basis and the purpose of scientific research. It requires agile organizations and it spells greater service and utility.

It is in this area of electronics of solids—the use of semi-conductors —that electronics engineers have an immediate future task. It is clear that the foundation stones of our industry are to be modified and changed. This is the challenge. The job is to move in an orderly but aggressive fashion. The job is to be sure that the advances are constructive and beneficial.

I have outlined what I consider today's Number One problem in military electronics—reliability. We have discussed what I consider today's Number One opportunity in peacetime electronics—television.

I have sketched what, in my opinion, is the Number One promise for electronics of tomorrow—electronics in the solid state. Shall we take one more step? Shall we attempt a glimpse at what, to me, seems to be the Number One promise for electronics the day after tomorrow?

I mentioned earlier that electronics, more and more, is assuming functions formerly performed by men's minds. Here today's elemental applications are prophetic for tomorrow and the day after tomorrow. In this area we face a revolution of techniques, mechanisms, and machinery—a revolution as great if not greater than any that has gone before. I cannot emphasize this too strongly.

Electronics with its unlimited ability to count, remember, and control—electronics with its capacity for accuracy—electronics with its comprehensive coverage—electronics with its facility to operate tirelessly and without fatigue—is electronics literally asking to take over certain duties which have been performed by men's minds—thinking processes. What man can conceive, comprehend and perform, he will be able to construct in electronic systems to do his bidding and the electronic performance will be at least as effective as the human performance.

I am not just thinking here of helps to man such as he has developed to aid his senses, to aid him to compute, to aid him to store information or a variety of other helps. I am thinking of a synthesis of all these together with developments yet to come, so organized that by man's pre-arrangement the electronic system will sense, react, interpret, compute, act and control. It will do this using what is the equivalent of thinking and intelligence. It will do this because man has built these characteristics into the system. It will do this at man's simple choice to start or to stop. It will act with such speed and with such all-inclusive comprehension that it will outstrip man in its action. It will permit solution of the most complex of situations.

How far will this go? Certainly it will include thinking processes which are repetitive. Certainly it will "think through" and execute wherever situations can be pre-analyzed and stored in electronic memory. Maybe this will include situations which today would be considered as creative thinking or at least in the border area of the creative. This may be so for material improvements, and also in the realm of the scientific and the humanistic—for all the arts and sciences.

Where will this stop? I for one do not now predict. But let me express a feeling—it is no stronger than a feeling. Electronic performance does have a limit, but it is a limit which advances with the continual progress made by man in his development of techniques and systems. While this is certainly an expanding horizon, these advances are linked and limited to the progress of man himself. Man's mind has no limit in fundamental creative thought and thus will always remain supreme.

I have discussed selected matters relating to the theme for this conference—Electronics for Defense and Industry. In closing let us think for just a moment of matters basic to the environments we shall find ahead. As we move through the years which separate us from one National Electronics Conference to the next, all individuals and all groupings of individuals in the free world will add their respective contributions, defining and determining the complicated paths of defense and peace. May those paths be directed so as to move away from the acts or even the threats of war. For our part, in our great country, our actions to preserve and guarantee peace depend in no small part upon

1. The faith we carry in our hearts—that faith which we propose, if necessary, to defend.
2. The vision we carry in our hearts—that vision which inspires us to uphold our faith and which will inspire others because of the vision we display.
3. The fervor we carry in our hearts—that fervor which carries us in acting our faith—actions to assure the dignity and freedom of the individual.

Faith — Vision — Fervor

These we need more than we may think!

A Diesel Engine Blower Test Stand in use at the Charleston Naval Shipyard makes it possible to test blowers under simulated shipboard operating conditions, discover oil leaks and overheating before blowers are installed in the ship. The stand was designed and built on the yard, fitted with a 165 hp Diesel. Blowers under test are operated at designed speed under three to six pounds back pressure for approximately six hours.

Army Reclaims Old Ammunition.—The Ordnance Ammunition Center of the U. S. Army at Joliet, Illinois, has purchased General Electric mouth-annealing equipment for installation at eight locations throughout the country in a dollar-saving program of reclaiming old ammunition of 40, 75, 76, and 90 millimeter calibers.

The purpose of the program is to reload shells that have become unreliable because of age. To reclaim the outdated shells, the projectiles and powder are removed from the cartridge cases and the mouth of the brass casing softened by induction annealing. This insures a snug fit against the grooves of the projectile when the cartridge cases are refilled and again crimped on the projectile.

Each of the eight installations will consist of a G-E 30-kw, 10,000 cycle motor-generator set with controls, and 2 two-position induction annealing machines which are designed to anneal cartridge case mouths at the rate of 600 per hour.

Two machines are used at each installation so one can cool and be reloaded while the other is going through the 11-second heating cycle, engineers said.

The machines are so constructed that the induction heating coils may be interchanged to accommodate shells of different caliber.

Near-Focusing Device for Close-up Photography.—E. Leitz, Inc., New York City, has announced a new near-focusing device which is designed to make it easier for amateurs, scientists, and professionals alike to take close-ups of manuscripts, pages from books, reference materials, and for recognizing photographs or slides. It also will prove useful to any photographer wishing to picture small objects such as flowers, small animals, or details of art objects which cannot be placed in a horizontal position. It is the Auxiliary Reproduction Device BOOWU, which can be used with all Leica Camera bodies and all of the 50 mm. Leica Lenses set at infinity.

The Auxiliary Reproduction Device is simple to adjust. It consists of three intermediate extension tubes and four extendable metal rods. Each tube is designed for a specific paper size: $8\frac{1}{4}\times11\frac{5}{8}$ in., $5\frac{7}{8}\times8\frac{1}{4}$ in., and $4\frac{1}{8}\times5\frac{7}{8}$ in. The user chooses the desired intermediate tube, screws the four rods into it, and pulls out the rods to the length required to frame the area. The ends of the rods circumscribe the size of the object and also determine the correct working distance.

The device can be set up or dismantled quickly. It is compact and easily can be carried in a brief case or accessory bag. It is ideal for libraries, scientific institutions, for use on field trips, and wherever close-ups are required.

NONLINEAR ANALYSIS OF ELECTRO-MECHANICAL PROBLEMS*

BY

Y. H. KU [1]

INTRODUCTION

The general expression for the study of electro-mechanical problems may be given in the following form:

$$\ddot{x} + f(\dot{x}, x) + f_1(x) = F(t), \tag{1}$$

where $\ddot{x} = d^2x/dt^2$, $\dot{x} = dx/dt$, x = angular or linear displacement, t = time, $F(t)$ = forced function which may consist of two parts: F_0 a constant and $F_1(t)$ a time function which is suddenly applied to the system. (In general, $F(t)$ can be expressed as $F_0(t) + F_1(t)1$, where the symbol 1 represents Heaviside's Unit Function signifying that $F_1(t)$ is suddenly applied at $t = t_0$.)

One of the important problems in an actual electro-mechanical system is to determine the actual maximum additional torque F_1 (or T_1 = constant) that can be suddenly applied to the system which has an original torque F_0 (or T_0 = initial constant load) within the stability limit. The same problem can be extended to the case where the original forced function is $F_0(t)$ and the added forced function is $F_1(t)$ beginning from $t = t_0$. In general, therefore, in Eq. 1, not only the functions $f(\dot{x}, x)$ and $f_1(x)$ are nonlinear, but also the function $F(t)$ may have discontinuities as the problem specifies.

This type of problem was first formulated by Lyon and Edgerton (1)[2] and solved by an integraph. Later on, differential analyzer solutions were obtained by Edgerton, Zak, Fourmarier, Brown and others at the Massachusetts Institute of Technology (2–4), and Lauder, Crary and Shoults of the General Electric Company (5), U. S. A. Analytical solution by nonlinear methods was first given by Keller (6). McLachlan (7) solved Lyon and Edgerton's problem for a non-salient synchronous machine by isocline construction (8) with the help of experimental knowledge from ref. 1. Stoker (9) discussed the same pull-out problem by a graphical method due to Liénard.

The method given in this paper tries to combine the merits of the differential analyzer and nonlinear solutions by a simple approach

* The substance of this paper was presented before the VIIIth International Congress of Theoretical and Applied Mechanics, Istanbul, Turkey, August, 1952, but the complete paper is here published for the first time.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

utilizing a new phase-plane plot involving the acceleration $\ddot{x}$ and the angular displacement (or linear displacement) x. From a plot of the acceleration-displacement curves for different values of $\dot{x}$, a new graphical method has been developed to construct velocity-displacement curves for different initial or boundary conditions. From the velocity-displacement curve, it is further possible to construct displacement-time curves. Assuming $F_0 = T_0 =$ constant and $F_1 = T_1 =$ another constant, and hence $F = T = T_0 + T_1 I$, the above method shows very clearly that starting from the $\ddot{x} - x$ plot on the acceleration plane, there is obtained the $\dot{x} - x$ plot on the velocity plane, and then the $x - t$ plot is readily made. The construction is much simpler than the isocline construction, as we need not construct all the isoclines, but only make the necessary construction for slope lines relevant to the given boundary conditions. It is more general than Liénard's method as $f(\dot{x}, x) + f_1(x)$ can be used instead of $\varphi(\dot{x}) + x$. In a particular electro-mechanical problem studied, it is found that the separatrix obtained from the new construction gives a definite set of information about stability limits so that the suddenly applied torque limit is readily obtained without going through the many trials (or runs) in either the differential analyzer or isocline solutions.

The method can be further extended to a forced function $F(t)$ as shown in Eq. 1. The method will be explained in connection with the solution of the Van der Pol Equation later:

$$\ddot{x} - k(1 - x^2)\dot{x} + x = F(t) \tag{2}$$

which can be considered as a special case of Eq. 1. In the following, it will be seen that the method is applicable to a more general type than the examples given which belong to a more specific type of the following form:

$$\ddot{x} + f(x)\dot{x} + f_1(x) = F(t). \tag{3}$$

THE GENERAL GRAPHICAL METHOD

From Eq. 1, let $v = \dot{x} = dx/dt$ and $F(t) = F_0 + F_1(t)$, and there is

$$\ddot{x} = F_0 + F_1(t) - f(v, x) - f_1(x) = G(v, x, t) = F_1(t) + g(v, x), \tag{4}$$

where G is a function of v, x, and t, and g corresponding to $G(v, x, F_0)$ is a function of v and x only. Notice that $F_1(t)$ can be generalized to $F_1(v, x, t)$ without altering the general form of the G function which represents acceleration values of $\ddot{x}$ for different values of v, x, and t. For $F_0 = 0$, the g function represents a characteristic of the system without regard to the form of forced function $F(t)$ or its initial value F_0 or $F_1(t_0)$.

We shall consider Eq. 4 for a given instant $t = t_1$, when $F(t_1) = F_0 + F_1(t_1) = T =$ a constant at that moment. Let $g(v, x)$ be redefined as equal to $G(v, x, T)$. We shall plot acceleration against displacement or $g - x$ curves for different values of $v = \dot{x}$. These equi-

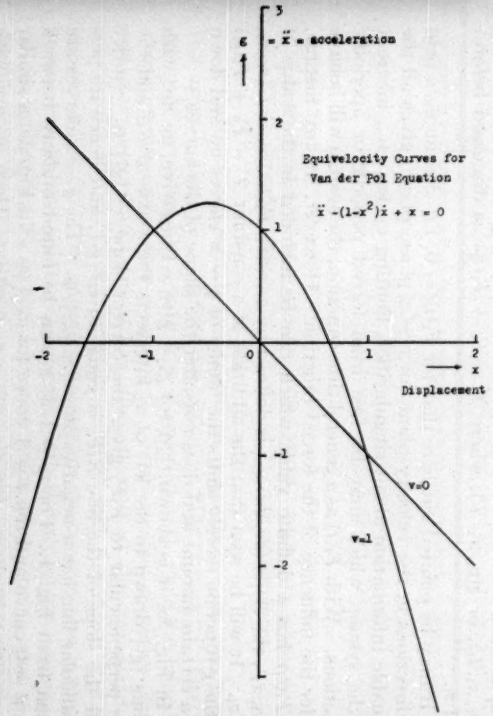

FIG. 1. Equivelocity curves on acceleration plane.

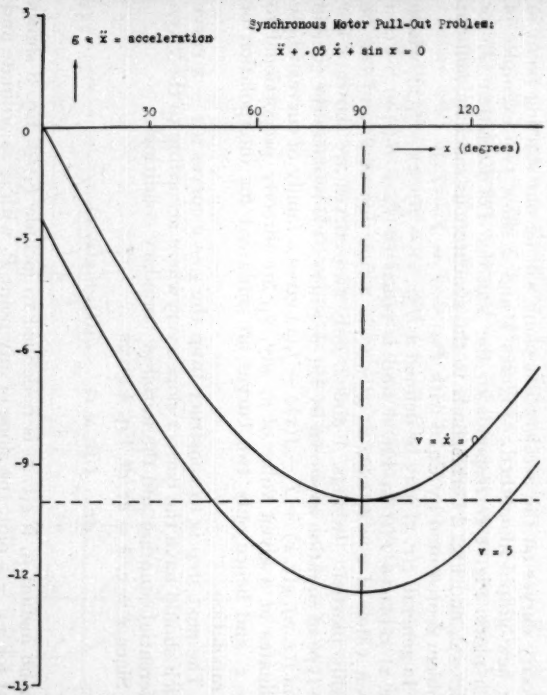

FIG. 2. Equivelocity curves on acceleration plane (g and v have scale factor 10).

velocity curves on the acceleration plane will be our starting point in the new graphical method. Figures 1 and 2 show two examples of such plots: Fig. 1 corresponds to the Van der Pol Equation (2) for $F(t) = 0$, and Fig. 2 corresponds to the synchronous machine pull-out problem represented by Eq. 3 with $F(t) = T = T_0 + T_1 1$.

In general, $g(v, x)$ may be defined as $F(v, x) - f(v, x) - f_1(x)$, where $F(v, x)$ replaces $F(t)$ in Eq. 1 and is equal to $F_0 + F_1(v, x)$. For a given value of $v = \dot{x}$, say, $\dot{x}_1$, $g(v_1, x)$ is a function of x, and can be readily plotted. In Figs. 1 and 2, only two curves are shown. For the type of equation as shown by Eq. 3, where $f(x)\dot{x}$ replaces the general form $f(\dot{x}, x)$, $g(v, x) = T - f(x)\dot{x} - f_1(x)$ gives a family of curves whose ordinates at a given value of x, say, x_1, are directly proportional to $v = \dot{x}$, and hence only two curves are sufficient for interpolation or extrapolation.

The next step is to construct from the $g - v$ curves a $v - x$ curve which should have the correct slope everywhere according to the given differential equation and the initial or boundary conditions.

Since $\dot{x} = v$, $\ddot{x} = dv/dt$, Eq. 4 gives

$$\frac{dv}{dx} = \frac{G(v, x, t)}{v} = \frac{F_1(t) + g(v, x)}{v}. \tag{5}$$

For an instant t_1, if $g(v, x)$ is re-defined to include $F_1(t_1)$, Eq. 5 shows that for a $v - x$ plot, the slope of any point P_1 with v_1 as ordinate and x_1 as abscissa is defined uniquely by the ratio of g_1 (corresponding to $G(v_1, x_1, t_1)$, or $g(v_1, x_1, T)$, where $T = F_0 + F_1(t_1)$ as discussed before) and v_1.

It may be pointed out here that for $F_1(t) = 0$, Eq. 5 defining a field of directions in the velocity plane (or $v - x$ phase plane) gives all the valuable information about singularities, limiting cycles, etc., inherent in the system which may be free from forced periodic or aperiodic functions. With $F_1(t)$ as a general time function, the system will behave under the influence of the forced function. However, at any instant, as $F_1(t_1)$ has a definite value which can be included in the value of $g_1(v_1, x_1)$, the slope $(dv/dx)_{v_1, x_1}$ is defined by $g_1(v_1, x_1, F_1(t_1))/v_1$ or $G(v_1, x_1, t_1)/v_1$. It will be seen that the addition of a constant $T = F_0 + F_1(t_1)$ to the g function merely shifts the family of $g - x$ curves up and down by a certain amount and does not alter the shape of the curves.

In Fig. 3*a*, it is shown that for $g_1 = g(v_1, x_1)$ negative, an arc with radius $|g_1|$ drawn to the left of x_1 intersects the x-axis at R_1, and the line perpendicular to R_1P_1 gives the correct slope $-|g_1|/v_1$. Notice that the slope of the line R_1P_1 is equal to $v_1/|g_1|$, and hence its perpendicular line has a negative slope $-|g_1|/v_1$. The $g - x$ curves are taken from Fig. 1. For $v_1 = 1/2$, g_1 can be found midway between the $v = 0$ curve and the $v = 1$ curve for $x = x_1$. In Fig. 3*b*, a similar construction is shown for $g(v_2, x_2) = g_2$ positive. Here an arc with

radius $|g_2|$ drawn to the right of x_2 intersects the x-axis at R_2. A line joining R_2 and P_2 (a point corresponding to v_2 and x_2 in the $v - x$ plane) has the negative slope $-v_2/|g_2|$ due to construction. A perpendicular drawn through P_2 hence has a positive slope $|g_2|/v_2$ which checks with Eq. 5 for $F_0 = 0$ and $F_1(t) = 0$. (As $v_2 = 1$, g_2 is taken from the $g - x$ curve for $v = 1$.)

The above-mentioned simple construction gives the correct slope everywhere in the $v - x$ plane based on direct readings from the $g - x$ plot. The correctness of the slope does not depend upon whether in taking increments of x and v the direction of Δx or Δv is positive or negative.

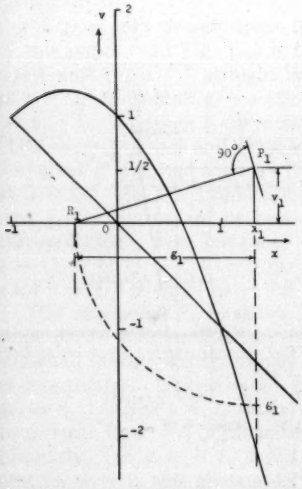

FIG. 3a. Construction of slope at P_1 on velocity plane.

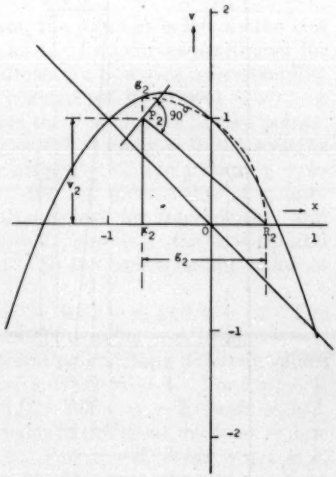

FIG. 3b. Construction of slope at P_2 on velocity plane.

A further step will be explained in connection with two typical examples whose $g - x$ curves are given in Figs. 1 and 2. The first example considers Eq. 2 with $k = 1$ and $F(t) = 0$. Let the boundary condition be given as $x = 2$, $v = 0$. (This specific condition is not necessary according to preceding discussion about slope construction. Physically, we may start from some point P' with $v = v'$ and $x = x'$ as shown in Fig. 5.) The general principle for the construction is to start from some known point which agrees with the actual operating point of the system like P_0 in Fig. 4 or from a singularity point such as the saddle point S in Fig. 9 for the other example. The construction may

not necessarily start from $t = 0$. It may start from $t = t_0$ or $t = t_1$ in the phase-plane plot, so long as one remembers what he is doing and where the plot starts. By starting from a saddle point S, one may be retracing his steps backward. However, since Eq. 5 gives the correct slope everywhere, one is free to start at any point which has a physical significance.

In both examples, we start from $v = v_0$ and $x = x_0$. We have chosen $v_0 = 0$ for the sake of simplicity. In the first example, the initial slope is $g/0 = \infty$, as $g = -x = -2$ for $x = 2$. In the other example, the initial slope is $0/0$ for $g = 0$ and $v = 0$ at $x = x_s$ corresponding to the saddle point S, and hence indeterminate. Consider a small interval $2(\Delta x)$ to the left of x_0 (or x_s) and assume a direction x_0P_2 or P_0P_2, either upward or downward, as shown in Figs. 4*a* and 4*b*. Mark the end of the interval $2(\Delta x)$ as x_2 and mark the mid-point as x_1, where $x_1 = x_0 - \Delta x$ according to construction. From x_1, erect a vertical line until it intersects the line P_0P_2 at P_1, which by construction has an ordinate v_1 (positive or negative) and an abscissa x_1. The slope at P_1 or at v_1, x_1 must agree with the correct slope $(dv/dx)_{v_1,x_1} = g_1/v_1$. If the assumed slope does not check with the construction shown in Fig. 3, the assumed slope must be duly modified.

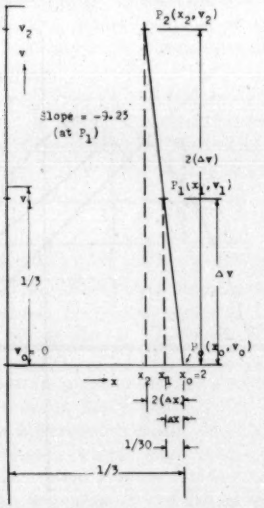

FIG. 4*a*. Construction of slope line from P_0 to P_1.

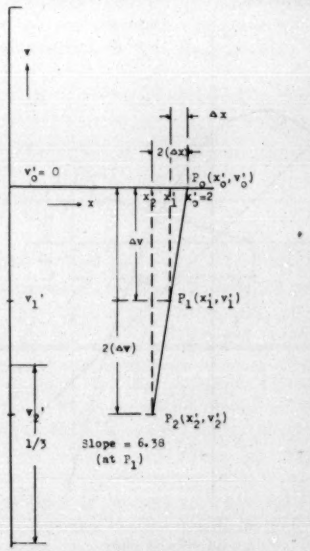

FIG. 4*b*. Construction of slope line from P_0 to P_1.

This initial slope from v_0, x_0 to v_2, x_2 giving the correct slope at v_1, x_1 can be checked analytically as follows: Since Δx is small, Eq. 5 gives

$$\frac{\Delta v}{\Delta x} = \frac{v_1 - v_0}{\Delta x} = \frac{g(v_1, x_1)}{v_1}.$$

SOLUTION OF VAN DER POL EQUATION

Consider the Van der Pol Equation as given by Eq. 2 with $F(t) = 0$. We have from Eq. 5,

$$\frac{dv}{dx} = \frac{k(1 - x^2)v - x}{v} = \frac{g(v, x)}{v}. \tag{6}$$

For $F(t) = \sin t$ or any other function, the solution is left to the end of this paper. In Fig. 1, k is taken as 1. Two curves are drawn for $v = 0$ and $v = 1$. A straight line is drawn for $g = -x$, corresponding to $v = 0$. A second $g - x$ curve is constructed for $g = (1 - x^2) - x$ for $v = 1$. The $v = 1$ curve intersects the $v = 0$ curve at two points: (1) $x = -1$, $g = 1$ and (2) $x = 1$, $g = -1$. For $x = 0$, the straight line gives $g = 0$, and the $v = 1$ curve gives $g = v$. (In general, $g = kv$ at $x = 0$.) As g is proportional to v for any given value of x, only two $g - x$ curves are necessary and sufficient for interpolation and extrapolation. For instance, at $x = 2$, $g = -2$ for $v = 0$, and $g = (1 - 4)(1) - 2 = -5$ for $v = 1$. So the corresponding value of g for $v = 1/2$ is equal to -3.5.

For any point P_1 as shown in Fig. 3*a*, let $v = v_1$ and $x = x_1$. The slope dv/dx is determined by Eq. 6 as $[(1 - x_1^2)v_1 - x_1]/v_1$ for $k = 1$. We shall investigate the slopes of several points along different values of coordinates. Take $x = 1$. For $v = 1$, $dv/dx = -1$. For $v = -1$, $dv/dx = 1$. For $v = 2$, $dv/dx = -1/2$. For $v = -2$, $dv/dx = 1/2$. In general, for $x = 1$ (k can have any value in this case), $dv/dx = -1/v$. Similarly, for $x = -1$, $dv/dx = 1/v$. For $x = 0$, $dv/dx = g/v = k$, and for $k = 1$, this gives a 45° slope for different values of v. Next take $x = 2$. For $v = 1$, $dv/dx = g/v = -5/1 = -5$. For $v = 2$, $dv/dx = -8/2 = -4$. (Notice that $g = -8$ for $x = 2$ and $v = 2$ can be extrapolated from $g = -2$ for $v = 0$ and $g = -5$ for $v = 1$ at $x = 2$.) Similarly, for $v = -1$, the slope corresponding to $x = 2$ is $dv/dx = g_{(v=-1)}/-1 = 1/-1 = -1$. For $x = -2$, the slope is given by $dv/dx = g/v = (-3v + 2)/v = -3 + 2/v$. For $v = 1$, this gives $dv/dx = -1$. For $v = -1$, this gives -5. For $v = 2$, the slope is $dv/dx = -2$; and, for $v = -2$, the slope is $dv/dx = -4$.

According to our method, it is not necessary to construct all these slope lines, although we can easily construct all such slope lines as in the isocline method. In fact, we can choose any starting point P_1, check its slope line, and proceed with the phase-plane construction as follows.

As mentioned before, just for the convenience of illustration, let

the starting point be chosen in the $v - x$ plot as corresponding to $v = 0$ and $x = 2$ for the solution of Van der Pol Equation as shown in Fig. 5. From Eq. 6, the slope dv/dx at point P_0 is given by $g/v = -2/0 = \infty$, for $F(t) = 0$. So the slope is vertical at the starting point. We can then proceed toward either positive values of v or negative values of v. In choosing the positive increments of v with decreasing increments of x, the incremental change in time is negative, and hence we are retracing

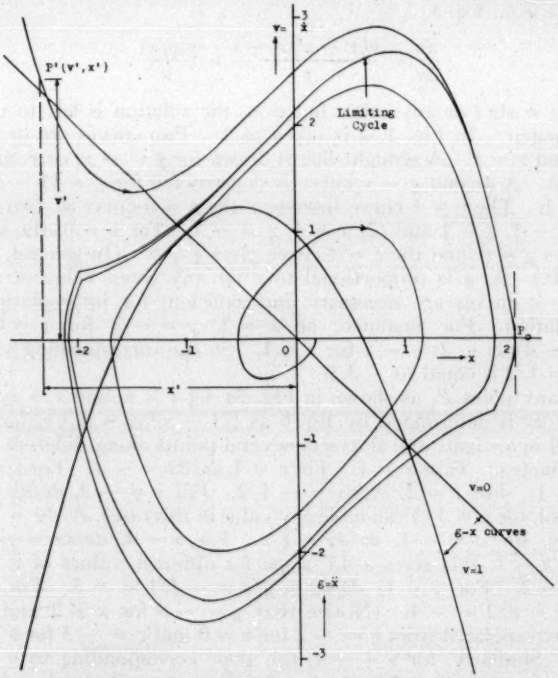

FIG. 5. Velocity-plane plot showing limiting cycle for Van der Pol equation: $\ddot{x} - (1 - x^2)\dot{x} + x = 0$.

our steps in the physical problem. Suppose we assume a certain upward slope as shown in Fig. 4*a* with a small increment $2(\Delta x)$ to the left of $x = x_0 = 2$. Let $2(\Delta x) = 1/15$. We need to check the slope at a point P_1 midway from x_0 or P_0 to a point P_2 which intersects the ordinate erected from $x_2 = x_0 - 2(\Delta x)$. Obviously, the mid-point P_1 corresponds to a chosen value $x_1 = x_0 - \Delta x$ and an assumed value $v_1 = (v_2 + v_0)/2$ as determined by the assumed slope from P_0 to P_2.

However, it is necessary to check the slope at v_1, x_1 to see if it is correct such that $(dv/dx)_{v_1,x_1} = g(v_1, x_1)/v_1 = g_1/v_1$.

Since $\Delta x = 1/30$, $x_1 = 2 - 1/30 = 59/30$. For $k = 1$, we get $g_1 = [1 - (59/30)^2]v_1 - 59/30 = -2.8677v_1 - 1.9667$, or $dv/dx = g_1/v_1 = -2.8677 - 1.9667/v_1$. As the increment is small (or it can be made as small as one pleases), we can replace approximately dv/dx by $\Delta v/\Delta x$ as follows:

$$\frac{\Delta v}{\Delta x} = \frac{v_2 - v_0}{2(\Delta x)} = \frac{v_1 - v_0}{\Delta x} = \frac{g_1}{v_1},$$

where $v_1 = (v_2 + v_0)/2$. Putting in numerical values, there is

$$\frac{\Delta v}{\Delta x} = \frac{v_1 - v_0}{(-1/30)} = \frac{g_1}{v_1} = -2.8677 - 1.9667/v_1$$

which gives $v_1 = 0.3085$. The slope at $v_1 = 0.3085$ and $x_1 = 1.9667$ is, according to Eq. 6,

$$\left(\frac{dv}{dx}\right)_{(v_1,x_1)} = \frac{g(v_1, x_1)}{v_1} = \frac{-2.852}{0.3085} = -9.25$$

which checks with $-30(v_1 - v_0) = -30(0.3085) = -9.255$.

Similarly, if we take a downward slope (Fig. 4*b*), the initial slope is given by $\Delta v/\Delta x = g_1/v_1 = -1.3567/-0.2125 = 6.38$, corresponding to $v_1 = -0.2125$.

Thus it is seen that for an initial condition of $v_0 = 0$ and $x_0 = 2$, a change of $x = 1/30$ to the left of x_0 corresponds to an increase from $v = 0$ to $v = 0.3085$ or a decrease from $v = 0$ to $v = -0.2125$. The slope corresponding to $v_1 = 0.3085$ is -9.25 and the slope corresponding to $-v_1' = -0.2125$ is 6.38. In fact, both values of v_1 at x_1 are given by the equation $v_1^2 = (2.868/30)v_1 + 1.967/30$, or $v_1 = 0.0478 \pm 0.2604 = 0.3082$ or -0.2126, and the corresponding slopes are given by $-30\ v_1$. So the values of v_1 or $\Delta v = (v_1 - v_0)$ corresponding to $x_1 = x_0 - \Delta x$ and their slopes at v_1, x_1 can be obtained either graphically or analytically. The analytical solution is in a way guided by the graphical construction and in turn helps to check the graphical solution.

In Fig. 5, consider the curve starting from P' corresponding to $x' = -x_0'$ and v' in the second quadrant. For an increase of Δx to the right, v first decreases and then increases. For $x = -1$, the slope is $1/v$; that is, the slope line is perpendicular to the line drawn from the origin to the point where the curve crosses the $x = -1$ ordinate. Then the curve passes the $x = 0$ ordinate or the vertical axis at 45 degrees positive slope, reaches a maximum at $x < 1$, crosses the $x = 1$ ordinate with a $-1/v$ slope, and then in a downward slope reaches the point $v = 0$, $x = 2$. From this point on, the velocity becomes negative and the curve follows the limiting cycle as shown.

ANOTHER FORM OF VAN DER POL EQUATION

We shall consider briefly another form of Van der Pol Equation as follows:

$$\ddot{x} - k(1 - \tfrac{1}{3}\dot{x}^2)\dot{x} + x = 0. \tag{7}$$

Differentiating Eq. 7 with respect to time, there is

$$\dddot{x} - k(1 - \dot{x}^2)\ddot{x} + \dot{x} = 0. \tag{8}$$

By substituting y for $\dot{x}$, we get back to the form given in Eq. 2.

Let $v = \dot{x}$ as before. Equation 7 becomes

$$\frac{dv}{dx} = \frac{k(1 - v^2/3)v - x}{v} = \frac{g(v, x)}{v}. \tag{9}$$

Here $g(v, x)$ may be separated into two parts: $\varphi(v)$ and $f_1(x)$. Notice that $\varphi(v)$ may take a more general form than that given in Eq. 9, say, $\varphi(v) = k(1 - av^2 + bv^4 + \cdots)v$. Similarly, $f_1(x)$ may consist of x and

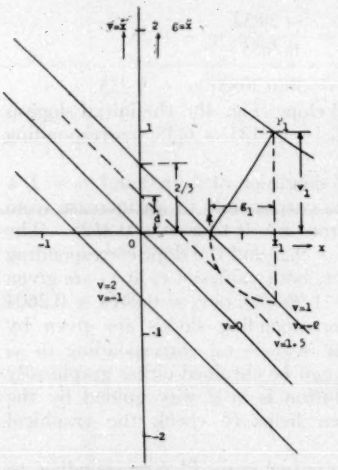

FIG. 6. $\ddot{x} - \dot{x} + \dot{x}^3/3 + x = 0$.

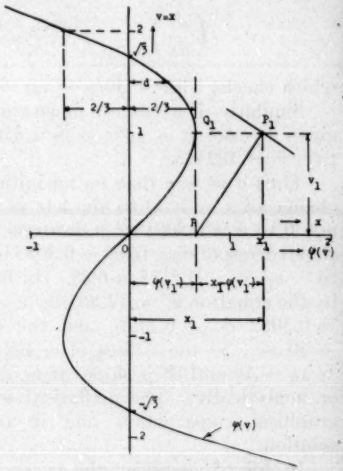

FIG. 7. Liénard construction.

higher powers of x or sin x and sin $2x$, etc. As, in general, $\varphi(v)$ is not simply kv, $\varphi(v)$ for different values of v must be considered or plotted as a part of $g(v, x)$. Similarly, $f_1(x)$ for different values of x can be calculated or plotted. Taking Eq. 9 as an illustration, since $\varphi(v)$ has a definite value for a given v, say, v_1, $g(v_1, x) = \varphi(v_1) - x$ is a straight line on the $g - x$ plane. In Fig. 6, three lines are shown: (1) for $v = 0$, the line represents $g = -x$, and $g = 0$ at $x = 0$; (2) for $v = 1$,

the line represents $g = 2/3 - x$ (assuming $k = 1$) giving $g = 2/3$ at $x = 0$ and $g = 0$ at $x = 2/3$; (3) for $v = 2$, the line represents $g = -2/3 - x$ giving $g = -2/3$ at $x = 0$ and $g = 0$ at $x = -2/3$. Obviously, these lines are parallel lines with a negative slope of 1 or 45°. For $v = -1$, the line coincides with the line for $v = 2$, and, for $v = -2$, the line coincides with the line for $v = 1$. An auxiliary curve for $\varphi(v) = (1 - v^2/3)v$ is shown in Fig. 7. In Fig. 6, for $v = 1\frac{1}{2}$, the line is shown as dotted. For $x = 0$, $g = \varphi(v)$ can be found from the auxiliary plot shown in Fig. 7. In Fig. 7, we have taken v as the vertical axis, and $\varphi(v)$ as the horizontal axis which coincides with the x-axis in the $v - x$ plot. In Fig. 6, we have shown the construction for finding the slope at P_1 corresponding to $v = 1$ and $x = 1\frac{1}{3}$. In Fig. 7, correlation of the new construction and Liénard's method is shown. It is evident that the two methods check with each other. However, Liénard's method cannot be readily extended to the case where instead of $\varphi(v)$, the function $f(v, x)$ involves both v and x and cannot be separated into $\varphi(v)$ and $f_1(x)$.

In Eq. 9, if $-x$ is replaced by $f_1(x)$, then for $v = 0$, $g = f_1(x)$, for $v = 1$, $g = 2/3 + f_1(x)$, for $v = 2$, $g = -2/3 + f_1(x)$. So on the $g - x$ plane, only one curve $f_1(x)$ needed to be plotted, while the intercepts on the g-axis are given by $\varphi(v)$ as shown in Fig. 7. Thus, in general, there will be a family of parallel curves instead of straight lines as shown in Fig. 6.

EQUATION OF A PENDULUM

Consider the simple equation of a pendulum as a preliminary to the solution of the pull-out of synchronous machines.

$$\ddot{x} + k\dot{x} + k_1^2 \sin x = 0. \tag{10}$$

For $k = 0$ and small values of x, Eq. 10 is sometimes given as

$$\ddot{x} + k_1^2(x - x^3/3!) = 0. \tag{11}$$

According to the method outlined before, there is

$$\frac{dv}{dx} = \frac{g(v, x)}{v} = \frac{-kv - k_1^2 \sin x}{v}. \tag{12}$$

For $v = 0$, $g = -k_1^2 \sin x$. This gives the general shape of the family of curves. For different values of v, the sine wave is shifted vertically up or down by the amount equal to kv.

For $k = 0$ and $k_1^2 = g/L$, where g = gravitational constant and L = length of pendulum, Eq. 10 reduces to

$$\ddot{\theta} + (g/L) \sin \theta = 0, \tag{13}$$

where θ = angular displacement of the pendulum and $\ddot{\theta} = d^2\theta/dt^2$.

Let $w = d\theta/dt$ = angular velocity. Then similar to Eq. 12, we have

$$\frac{dw}{d\theta} = \frac{g(\theta)}{w} = \frac{-(g/L)\sin\theta}{w} = \frac{-k_1^2 \sin\theta}{w}. \tag{14}$$

This gives, for $k_1 = \sqrt{g/L}$,

$$w\,dw + k_1^2 \sin\theta\,d\theta = 0 \tag{15}$$

which can be integrated to get

$$w^2 = 2k_1^2 \cos\theta + \text{constant } A. \tag{16}$$

The initial condition $w = w_0$ and $\theta = \theta_0$ gives

$$A = w_0^2 - 2k_1^2 \cos\theta_0.$$

So the constant A depends on the initial values w_0 and θ_0. Equation 16 then gives

$$w^2 = w_0^2 + 2k_1^2(\cos\theta - \cos\theta_0). \tag{17}$$

The above discussion can be easily extended to a general form of $g(\theta)$ or $f(x)$ in Eqs. 14 and 10.

From Eq. 16, we get an analytical expression for the relation between w and θ, similar to the relation between v and x in our $v - x$ plot by the proposed graphical method. It is evident that we shall get a family of curves in the $v - x$ or $w - \theta$ plot for different values of constant A, which in turn depends on the initial values of v and x or w and θ. For $k_1^2 = 1/2$, Eq. 16 gives $w^2 = \cos\theta + A$ or $v^2 = \cos x + A$. The plot is shown in Fig. 8.

There are three sets of plots in Fig. 8: (1) 0-curve with P_0 as a representative point on the $v - x$ curve; (2) 1-curve with P_1 as a representative point on the $v - x$ plot; and (3) 2-curve with P_2 as its representative point. The P_0 curve corresponds to an initial condition $v_0 = 0$ and $x_0 = 90°$ so that $\cos x_0 = 0$ and $A = 0$. The P_1 curve corresponds to an initial condition $v_0 = 0$ and $x_0 = 60°$ so that $\cos x_0 = 0.500$ and $A = -0.500$. The P_2 curve corresponds to an initial condition $v_0 = 0$ and $x_0 = 30°$ so that $\cos x_0 = 0.866$ and $A = -0.866$. For $v_0 \neq 0$, the values of A will be changed by the constant v_0^2. Three dotted curves for Eq. 16 are shown with w^2 or v^2 as the ordinate and θ or x as the abscissa. These dotted curves cross the horizontal axis at 90°, 60°, and 30°, respectively. (These points are shown on the θ or x-axis which is scaled according to radians.) It may be noted that the w^2 or v^2 curves represent kinetic energy curves of the system. The v-ordinate at any value of x (or θ) can be readily obtained if $w^2 - \theta$ or $v^2 - x$ plot is known. In order to check our general method from the acceleration-plane plot, let us look at the $g - x$ curve which is, according to Eq. 14, $g(x) = -k_1^2 \sin x = -1/2 \sin x$. Since $g(v, x)$ is simplified to $g(x)$, a function of the displacement only, the $g - x$ plot

is valid for all values of v. Take the P_1 curve for example. For $x = x_1 = 1$ radian, $g = g_1$ as shown. From the point $x_1 = 1$, draw an arc with radius equal to g_1 and cut the x-axis to the left of x_1 at a point R_1. From R_1, draw a line to P_1, and the perpendicular line to R_1P_1 will give the correct slope $dv/dx = g_1/v_1$. Similarly, for a point P_2 on

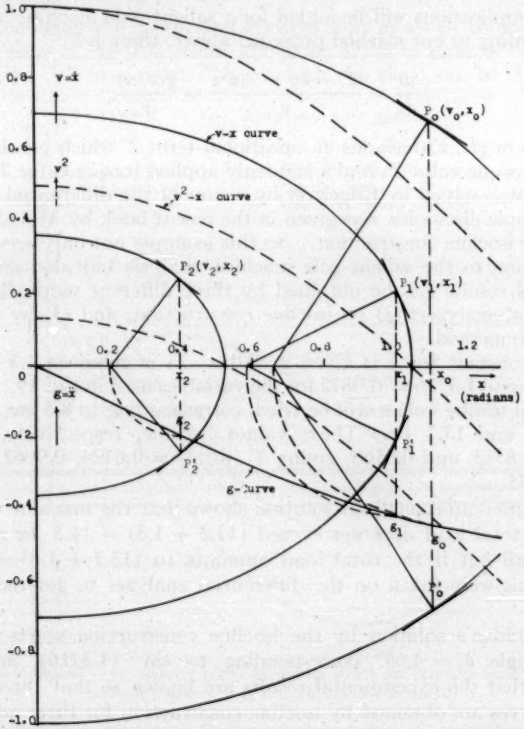

FIG. 8. Velocity-plane plot for a pendulum. $\ddot{x} + k_1^2 \sin x = 0$. $(k_1^2 = \frac{1}{2})$.

the P_2 curve, $x_2 = 0.4$, $g = g_2$. From R_2, which is g_2 distant from x_2 as shown, draw a line joining the assumed point P_2. Then the correct slope can be checked from the relation given in Eq. 12 or Eq. 14.

THE SYNCHRONOUS MACHINE PROBLEM

The pull-out problem of a synchronous machine can be expressed as a nonlinear differential equation as follows:

$$\ddot{x} + k\dot{x} + \sin x = T = T_0 + T_1 H(t), \tag{18}$$

where $\ddot{x}$ represents the inertia torque, $k\dot{x}$ represents the damping torque, $\sin x$ represents the synchronous motor torque, T = shaft torque (total), T_0 = initial torque, and T_1 = amount of torque suddenly applied to the system, with $H(t)$ or 1 signifying Heaviside's Unit Function. As it was discussed in Lyon & Edgerton's paper (1), the above form can be obtained by suitable change of variables. (Additional complications will be added for a salient-pole machine later.)

According to our method proposed above, there is

$$\frac{dv}{dx} = \frac{T - kv - \sin x}{v} = \frac{g(v, x)}{v}. \tag{19}$$

Notice here $g(v, x)$ includes an additional term T which consists of an original torque value T_0 and a suddenly applied torque value T_1. This problem was solved by Edgerton by means of the differential analyzer and a simple discussion was given in the recent book by McLachlan (7) using the isocline construction. So this example not only serves as an introduction to the salient-pole machine analysis but also shows how numerical results can be obtained by three different methods: (1) by differential analyzer; (2) by isocline construction; and (3) by acceleration-plane method.

The constant for k is given as 0.05. T_0 is given as 1.3 *kw*, corresponding to $1.3/16 = 0.0812$ for the variables used in Eq. 19. Several additional torque values will be tried, corresponding to 8.5 *kw*, 13.2 *kw*, 13.3 *kw*, and 13.7 *kw*. These values become, respectively, 0.5312, 0.8250, 0.8313, and 0.8563, giving T (total) = 0.6124, 0.9062, 0.9125, and 0.9375.

The differential analyzer solution shows that the machine will stay in if the total load does not exceed $(13.2 + 1.3) = 14.5$ *kw* and that it will pull out if the total load amounts to $(13.7 + 1.3) = 15$ *kw*. Many runs were taken on the differential analyzer to get these early results.

McLachlan's solution by the isocline construction starts with an initial angle $\theta_0 = 4.66°$ corresponding to $\sin^{-1}(1.3/16)$, and then assumes that the experimental results are known so that three sets of $v - x$ curves are obtained by isocline construction for three additional loads corresponding to 8.5 *kw*, 13.2 *kw*, and 13.7 *kw*. During the supervision of a thesis by Rui de Figueiredo, the writer imposed on him the following requirement. Given a synchronous machine with mechanical and electrical constants (which can be measured), it is required to find out the limiting total torque and additional torque corresponding to a given initial torque, without any previous knowledge from either experiment or differential analyzer solution.

The following techniques are developed using the acceleration-plane plot. As shown in Fig. 2, the $g - x$ curves specify the behavior of the

synchronous machine according to Eq. 19 at $T = 0$. By shifting the horizontal axis, we have a family of $g - x$ curves corresponding to different values of total T. In Fig. 9, T is taken as 0.9125 corresponding to an additional torque load of 13.3 *kw* or a total torque load of 14.6 *kw*. The initial angle is shown as 5° or 4.66° corresponding to an additional load of 1.3 *kw*.

By following the method of construction as given before, one can start at $x_0 = 5°$, $v_0 = 0$, where the slope $dv/dx = g_0/v_0 = -0.0812/0$ = infinite. When a point P is reached, the slope can be checked by

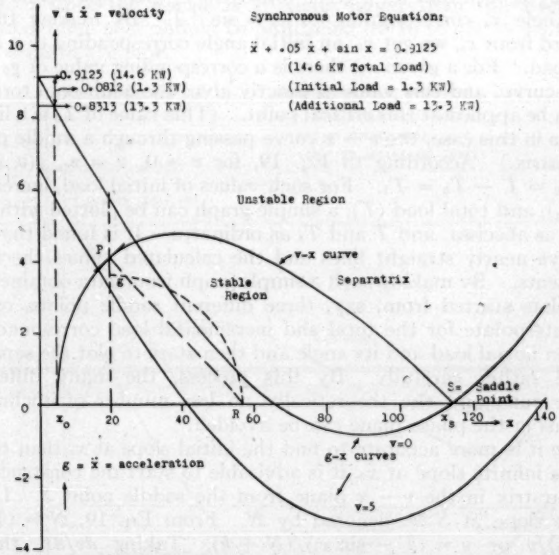

FIG. 9. Separatrix on velocity plane separating stable and unstable regions in the pull-out phenomena (g and v have scale factor 10).

$dv/dx = g_1/v_1$, as shown. The $v - x$ curve reaches a point x_s, where v is again zero, but the slope dv/dx is neither zero nor infinite, but indeterminate. From Eq. 19, this point x_s corresponds to $dv/dx = g/v = 0/0$, for, since $v = 0$ by plot on the $v - x$ plane, g must also be zero. For a given total torque T, this means that $g(v, x) = T - k(0) - \sin x_s = 0$, or $T = \sin x_s$. Hence in our example chosen, $T = 0.9125$, and $x_s = \sin^{-1} 0.9125 = 114.2°$. This point is marked S, signifying that it is a saddle point. The $v - x$ curve is a separatrix, as it separates the stable region and the unstable region. For any load greater than this critical load, the machine will pull out. For any load

smaller than this load, the machine will stay in and settle to a given stable angle of operation.

As shown in Fig. 9, on the $g - x$ curve for $v = 0$, g at $x = x_0 = 4.66°$ gives $-\sin x_0 = -0.0812$, or is equal to $-T_0$, measured from the horizontal line drawn through the intersection of $v = 0$ g-curve and the vertical axis. As the total distance between this horizontal line and the x-axis line is equal to $T = 0.9125$, the ordinate erected from x_0 to the $v = 0$ g-curve gives the additional load $T_1 = 0.8313$ corresponding to 13.3 *kw*.

It can be seen that for a given total load T, there is a saddle point S at an angle x_s corresponding to $x_s = \sin^{-1} T$. By making the plot backward from x_s, we get x_0, an initial angle corresponding to a certain initial load. For a given x_0, there is a corresponding value of g_0 on the $v = 0$ g-curve, and this value g_0 exactly gives the additional torque T_1 that can be applied at this critical point. (This value of T_1 is a limiting value, as in this case, the $v - x$ curve passing through a saddle point is a separatrix.) According to Eq. 19, for $v = 0$, $x = x_0$, $g(v, x) = T - \sin x_0 = T - T_0 = T_1$. For such values of initial load, incremental load (T_1), and total load (T), a simple graph can be plotted with initial load T_0 as abscissa, and T and T_1 as ordinates. It is found that these plots give nearly straight lines and the calculated values check with experiments. By making such a simple graph from data obtained from $v - x$ plots started from, say, three different saddle points, one can easily interpolate for the total and incremental load corresponding to the given initial load and its angle and then start to plot the separatrix required rather carefully. By this process, the many differential analyzer runs and the theoretically no less number of isocline constructions in the phase plane can be avoided.

Since it is more accurate to find the initial slope at x_s than to start from the infinite slope at x_0, it is advisable to start the construction of the separatrix in the $v - x$ plane from the saddle point S. Let the negative slope at S be denoted by N. From Eq. 19, $N = (T - kv - \sin x_s)/v$ or $v = (T - \sin x_s)/(N + k)$. Taking dv/dx, there is $(dv/dx) = N = -\cos x_s/(N + k)$. Thus the slope N is

$$N(\text{slope}) = -(k/2) \pm \sqrt{(k/2)^2 - \cos x_s}, \tag{20}$$

where the negative sign is to be taken.

In Eq. 18 for the synchronous motor, if x is given in radians and v is given in radians per unit of time (the unit of time has been changed from t in seconds to $t' = at$ in the original reference), k should be $\sqrt{57.3}$ times larger than 0.05 as discussed in refs. 1 and 2. Equation 20 given above is strictly correct for x in radians, v in radians per unit time, $k = \sqrt{57.3} \times 0.05$ and N follows the same way. If we use $k = 0.05$, Eq. 20 must be modified to

$$N(\text{slope}) = -(k/2) - \sqrt{(k/2)^2 - (\cos x_s/57.3)}$$

corresponding to x in degrees and v in degrees per unit time used in Eq. 18. It may also be pointed out that in Fig. 9, besides the use of degrees for x, we have increased both the g-scale and the v-scale by 10 times. However, as the slope at any point depends on g/v, increasing both the g-scale and the v-scale by the same amount will not alter the true slope.

Coming back to our numerical example, there is

$$N = -\frac{0.05}{2} - \sqrt{\left(\frac{0.05}{2}\right)^2 + (0.410/57.3)} = -0.1132.$$

In Fig. 9, since the v-scale is 10 times larger than the x-scale, this calculated value of N should be multiplied by 10 to get -1.132 as the actual slope in the plot.

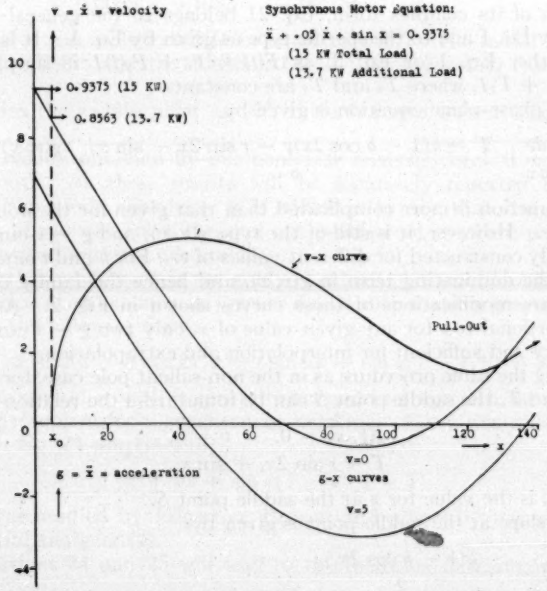

FIG. 10. Velocity-plane plot showing pull-out phenomena of synchronous machines.

Figure 10 shows the pull-out phenomena of the same synchronous machine when an additional torque load corresponding to 13.7 *kw* is suddenly applied in addition to the initial load of 1.3 *kw*. It is shown in Fig. 10 that if one starts from $x_0 = 4.66°$, the trajectory will never settle to a stable operating point, as it runs beyond the separatrix, never gets down to $v = 0$ again, and shoots off after it dips

near the saddle point. It may be of interest to note that for the total torque load corresponding to $(13.7 + 1.3) = 15$ *kw*, $T = 0.9375$ and $x_s = 110.43°$. So the saddle point is shifted about 4° to the left of 114.2°, and the slope will be $-0.1070 \times 10 = -1.070$. By starting from this new saddle point, one may get a new set of limiting conditions which, however, do not fit in with the physical requirements: $T_0 = 1.3$ *kw* and $T_1 = 13.7$ *kw*.

THE SALIENT-POLE SYNCHRONOUS MACHINE

For a salient-pole synchronous machine, Eq. 18 should be modified to the following:

$$\ddot{x} + k(1 - b\cos 2x)\dot{x} + r\sin 2x + \sin x = T = T_0 + T_1 H(t). \quad (21)$$

In spite of its complex form, Eq. 21 belongs to the general type as given by Eq. 1 and to the specific type as given by Eq. 3. It is simpler than either Eq. 1 or Eq. 3, as $F(t) = F_0 + F_1(t)I$ is simplified to $T = T_0 + T_1 I$, where T_0 and T_1 are constants.

The phase-plane equation is given by

$$\frac{dv}{dx} = \frac{T - k(1 - b\cos 2x)v - r\sin 2x - \sin x}{v} = \frac{g(v, x)}{v}. \quad (22)$$

The g function is more complicated than that given for the non-salient machine. However, it is still of the type $g(v, x)$, and $g - x$ curves can be readily constructed for different values of v. For k and r small, $\sin x$ is still the dominating term in $g(v, x)$, and hence the family of $g - x$ curves are modifications of those curves shown in Fig. 2. As $g(v, x)$ is proportional to v for any given value of x, only two $g - x$ curves are necessary and sufficient for interpolation and extrapolation.

Using the same procedure as in the non-salient pole case, for a given total load T, the saddle point S can be found from the relation:

$$g(v, x) = 0, \qquad v = 0,$$
$$T = r\sin 2x_s + \sin x_s,$$

where x_s is the value for x at the saddle point S.

The slope at the saddle point is given by

$$N(\text{slope}) = -\frac{k(1 - b\cos 2x_s)}{2} \pm \sqrt{\left[\frac{k(1 - b\cos 2x_s}{2}\right]^2 - 2r\cos 2x_s - \cos x_s}. \quad (23)$$

As discussed before, if x is expressed in degrees and v = degrees per unit time, the terms $(-2r\cos 2x_s - \cos x_s)$ should be divided by 57.3.

Equations 22 and 23 reduce to Eqs. 19 and 20 for $b = 0$ and $r = 0$.

For an actual machine studied by Rui de Figueiredo under the

writer's supervision, the constants are as follows:

$$k = 0.0068, \qquad b = 1, \qquad r = 0.338.$$

The variable t used or t' is equal to 0.0142 times the actual time in seconds. The total load T is taken as 1.0.

For a sample calculation, there is, for the saddle point,

$$T = 1.0 = 0.338 \sin 2x_s + \sin x_s$$
$$x_s = 90°.$$

The initial slope is given by

$$N = -\frac{0.0068(1+1)}{2} - \sqrt{(0.0068)^2 + \frac{0.676 - 0}{57.3}}$$
$$= 0.0068 - 0.1088 = -0.1156.$$

By construction, x_0 is found to be equal to 16.5°. The initial load T_0 is equal to 0.465 and the additional load (limiting value) is equal to 0.535.

Besides the saddle point at 90°, there is found a focal singularity at 42.5°.

The results obtained by our nonlinear analysis checked well with experiments. As these results will be separately reported by Mr. Rui de Figueiredo elsewhere, it is sufficient to say that the method outlined above can solve the pull-out problem of a salient-pole synchronous machine very adequately.

A further refinement in the study of salient-pole machines has been the addition of an exponential term to Eq. 21 such that the new equation assumes the following form:

$$\ddot{x} + k(1 - b\cos 2x)\dot{x} - r\sin 2x + \sin x(1 - \epsilon^{-at}) = T. \quad (24)$$

The term $\sin x(1 - \epsilon^{-at})$ is generally accepted to represent the pulling-into-step problem for a synchronous machine. For a non-salient-pole machine, Eq. 24 simplifies to:

$$\ddot{x} + k\dot{x} + \sin x(1 - \epsilon^{-at}) = T \quad (25)$$

which was studied by Edgerton, Brown, and others by means of the differential analyzer (4).

Equations 24 and 25 will lead to the following form according to Eq. 5:

$$\frac{dv}{dx} = \frac{G(v, x, t)}{v} = \frac{\sin x(\epsilon^{-at}) + g(v, x)}{v}. \quad (26)$$

GRAPHICAL SOLUTION INVOLVING FORCE FUNCTION $F(t)$

Take the Van der Pol Equation with a force function $F(t)$:

$$\ddot{x} - k(1 - x^2)\dot{x} + x = F(t) = \sin t. \quad (27)$$

Notice that we have taken the magnitude of sin t as 1 which is rather large, as usually the magnitude is taken as $\epsilon \ll 1$. As before, k is taken as 1, though it is also usually taken as $k \ll 1$ in analytical studies.

Equation 27 gives the following relation according to Eq. 5:

$$\frac{dv}{dx} = \frac{G(v, x, t)}{v} = \frac{F(t) + g(v, x)}{v} = \frac{\sin t + (1 - x^2)v - x}{v}. \quad (28)$$

At $t = t_0 = 0$, let us start with the initial condition $x = 2$ and $v = 0$ as in the previous example. However, a new function $F(t) = \sin t$ is to be introduced, so that at any point P_1, the correct slope dv/dx is now given by $G(t_1, x_1, v_1)/v_1$ instead of $g(x_1, v_1)/v_1$. Denoting the initial quantities as x_0, v_0, and t_0, we take a small increment $2(\Delta x)$ and assume a slope from x_0 to x_2 at the end of $2(\Delta x)$. Then consider the point P_1 mid-way between P_0 and P_2 as shown in Figs. 3 and 11. (As before, P_1 is a point with a given value of $x = x_1$ and an assumed value of $v = v_1$.) Since $\sin t_0 = 0$, we can calculate v_1 by the relation:

$$\frac{\Delta v}{\Delta x} = \frac{v_1 - v_0}{\Delta x} = \frac{\sin t_0 + (1 - x_1^2)v_1 - x_1}{v_1} \text{ (approx.)}$$

or

$$\frac{v_1 - 0}{-1/30} = \frac{0 + [1 - (59/30)^2]v_1 - (59/30)}{v_1} \text{ (approx.)}$$

which gives $v_1 = -0.2125$. The minus value is used in order to get the correct time sequence, as we want to proceed from $t_0 = 0$ to positive values of t, say, t_1, t_2, etc. Notice that it is not necessary that v_0, t_0, and $F(t_0)$ are zero as our starting point. In general, there is, for our first approximation of v_1,

$$\frac{v_1 - v_0}{\Delta x} = \frac{F(t_0) + g(x_1, v_1)}{v_1} \text{ (approx.)}.$$

Since for a chosen interval of Δx, all quantities are known except v_1, v_1 can be obtained either graphically or analytically. For our example, v_1 was obtained before as equal to -0.2125 corresponding to a positive slope 6.375 (or 6.38).

As $\Delta x = v\Delta t$, where v is the average velocity during the small interval, there is

$$\Delta t = \frac{-1/30}{(v_0 + v_1)/2} = \frac{2}{0.2125 \times 30} = \frac{2}{6.375} = 0.3137$$

and

$$t_1 = t_0 + \Delta t = 0 + 0.3137 = 0.3137.$$

The force function $F(t) = \sin t$ gives

$$F(t_1) = \sin t_1 = \sin 0.3137 = 0.3085.$$

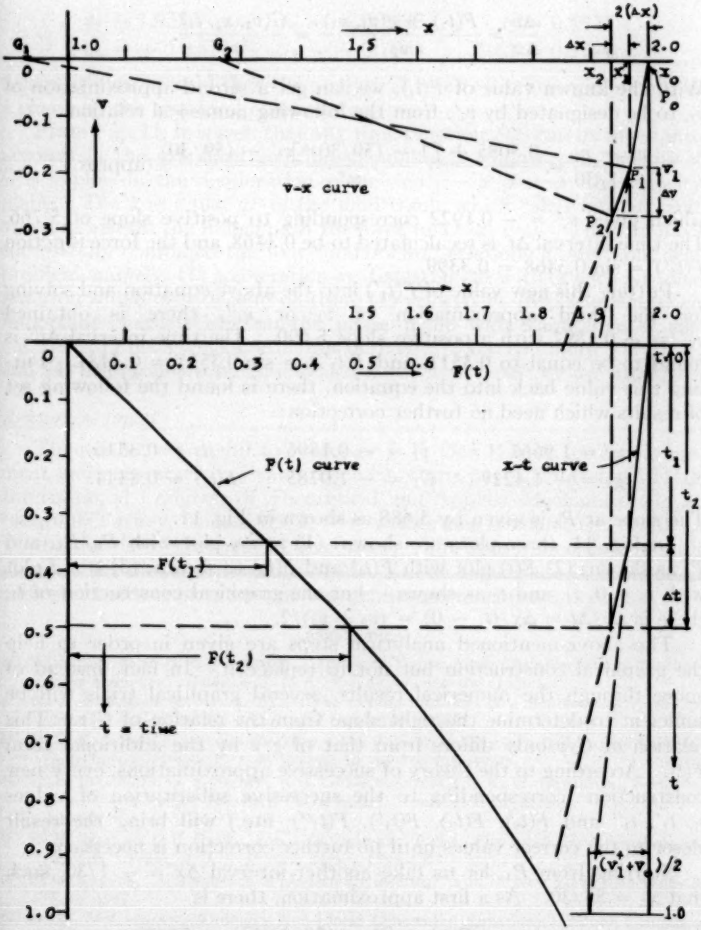

FIG. 11. Construction of $v - x$ curve and $x - t$ curve with forced function $F(t)$.

Going back to Eq. 28, we get, for small increments of Δv and Δx,

$$\frac{\Delta v}{\Delta x} = \frac{F(t_1) + g(v_1, x_1)}{v_1} = \frac{G(v_1, x_1, t_1)}{v_1}.$$

With the known value of $F(t_1)$, we can get a second approximation of v_1, to be designated by v_1', from the following numerical relation:

$$\frac{v_1' - v_0}{-1/30} = \frac{0.3085 + [1 - (59/30)^2]v_1' - (59/30)}{v_1'} \text{ (approx.)}$$

which gives $v_1' = -0.1922$ corresponding to positive slope of 5.766. The time interval $\Delta t'$ is recalculated to be 0.3468, and the force function $F(t_1') = \sin 0.3468 = 0.3399$.

Putting this new value of $F(t_1')$ into the above equation and solving for the third approximation of v_1 or v_1'', there is obtained $v_1'' = -0.1897$ with a positive slope 5.690. The time interval $\Delta t''$ is found to be equal to 0.3511, and $F(t_1'') = \sin 0.3511 = 0.3434$. Putting this value back into the equation, there is found the following set of results which need no further correction:

$$x_1 = 1.9667 \qquad v_1 = -0.1896 \qquad t_1 = 0.3516$$
$$g_1 = -1.4229 \qquad G_1 = -1.0785 \qquad F(t_1) = 0.3444.$$

The slope at P_1 is given by 5.688 as shown in Fig. 11.

In Fig. 11, three plots are shown: (1) $v - x$ plot with P_0, P_1, and P_2 as shown; (2) $F(t)$ plot with $F(t_1)$ and $F(t_2)$ as shown; (3) $x - t$ plot with $t_0 = 0$, t_1, and t_2 as shown. For the graphical construction of t_1, there is $\Delta x/\Delta t = \Delta x/(t_1 - 0) = (v_0 + v_1)/2$.

The above-mentioned analytical steps are given in order to help the graphical construction but not to replace it. In fact, instead of going through the numerical results, several graphical trials will be sufficient to determine the right slope from the relation of G/v. This relation of G/v only differs from that of g/v by the additional term $F(t)$. According to the theory of successive approximations, every new construction (corresponding to the successive substitution of values t_1, t_1', t_1'' and $F(t_0)$, $F(t_1)$, $F(t_1')$, $F(t_1'')$, etc.) will bring the result closer to the correct values until no further correction is necessary.

Starting from P_1, let us take another interval $\Delta x = -1/30$, such that $x_2 = 58/30$. As a first approximation, there is

$$\frac{v_2 - v_1}{-1/30} = \frac{F(t_1) + [1 - (58/30)^2]v_2 - 58/30}{v_2},$$

where $t_1 = 0.3516$ and $v_1 = -0.1896$. Going through successive approximations as before, there is obtained the following set of results

for $P_2(v_2, x_2)$ with a positive slope 2.54:

$$x_2 = 1.9333 \qquad v_2 = -0.2743 \qquad t_2 = 0.4955$$
$$g_2 = -1.1817 \qquad G_2 = -0.7063 \qquad F(t_2) = 0.5754.$$

Again, it may be said that the analytical results are here given only as a check on the graphical construction.

From Fig. 11, it is seen that any time function $F(t)$ can be taken into account in the graphical solution of nonlinear equations by means of $G - x$ plot on the acceleration plane and $v - x$ plot on the velocity plane. The $x - t$ plot gives the final result which takes into account at every instant the influence of the force function $F(t)$. Thus we have successfully combined the four fundamental relations in any nonlinear problem, namely: (1) acceleration *vs.* displacement with or without force function which may depend on time; (2) velocity *vs.* displacement as an important characteristic on the phase plane with knowledge of singularities, limiting cycles, etc. thus derived; (3) forced function *vs.* time; and (4) displacement *vs.* time which is the final result we are after.

Acknowledgment

The author is grateful to Professor J. P. Den Hartog for encouragement and presentation of the substance of this paper before the VIIIth International Congress of Theoretical and Applied Mechanics held at Istanbul, Turkey, August, 1952. He wishes also to thank Dr. Frank L. Wattendorf for his interest in this paper. The author records here with pleasure the fine contributions of Mr. Rui de Figueiredo in connection with the analysis of salient-pole synchronous machines.

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Vermiculite Insulating Firebrick.—An insulating firebrick that increases furnace efficiency and helps reduce operating costs has been introduced by the Zonolite Company, Chicago. The brick, made of vermiculite and clay binders, has extremely low thermal conductivity, which prevents excess heat loss in constant or intermittent furnace operations, with a resulting saving in fuel costs. Vermiculite, the micaceous mineral produced by Zonolite, is the key ingredient and provides the efficient insulating quality of the brick, according to Dr. George Ziegler, the firm's Director of Research.

Extensive testing by Zonolite shows that the bricks have high resistance to cracking caused by impact or thermal shock, have good physical and chemical stability within their temperature range, and are easily sawed and fitted without breakage, Dr. Ziegler said.

The brick is designed primarily as back-up insulation for refractory brick in the service range up to 1800 degrees Fahrenheit. When not subjected to mechanical abrasion, or exposed to molten slag or metal, however, it can be used as facing brick when temperatures are not higher than 1800 degrees. The bricks are made in standard 9-in. series, and are packed in cartons of 52 each. Any good refractory mortar can be used with them.

The physical properties of the new firebrick are the following:

Maximum service temperature	1800 F.
Density in pounds per cubic foot	35
Weight per brick	2.1 lb.
Thermal conductivity	1.25 at 700 F. mean
Cold crushing strength in pounds per square inch	185
Modulus of rupture in pounds per square inch	145
Percent of linear shrinkage after reheating to 1600 F.	0.1
Percent of linear shrinkage after reheating to 1800 F.	2.0

Heated Knives Cut Rayon Strip Without Fraying.—The problem of edge-fraying when cutting acetate rayon into strips has been eliminated by the Cameron Machine Company of Brooklyn, N. Y., through the use of General Electric Calrod* tubular heaters on the cutting knives of a recently announced slitting machine.

The straight, electrically heated blades of the new machine melt razor-thin slits in the rayon as the fabric passes between the blades and the rewinder platen roll. While cutting, the blades fuse minute strips along the edges of the cuts to prevent separation or beading. The edges cool instantly, according to Cameron engineers, and there is no danger of fused layers in the rewound rolls.

Slitter units operate efficiently at speeds up to 250 ft. per minute and can be set to slit to any width from $\frac{1}{4}$ in. up, across the full width of the material, the engineers said.

* Reg. Trademark of the General Electric Company.

THEORY OF THE PHOTIC FIELD

BY

PARRY MOON[1] AND DOMINA EBERLE SPENCER[2]

By photic field theory, we mean the application of vector field theory to the calculation of amounts of radiant power or of light. The subject is not electromagnetic field theory or physical optics. Rather, it is a branch of geometric optics. The wave nature of light—all such aspects as diffraction and polarization—are ignored, and attention is concentrated on radiant power and its distribution. For many purposes, this approach effects a great simplification and gives important practical results very simply.

The classical methods of calculating light were developed in the eighteenth century by Lambert and by Bouguer, and these elementary methods are still in use. The first attempt to apply vector theory to the subject seems to have been made by Mehmke (1)[3] in 1893. Since then, a number of advances have appeared in photic field theory and in its applications to practical problems. There is a big difference, however, between the mere addition of vectors and the comprehensive theory of fields. Most authors have confined themselves to the elementary vector algebra of the photic field. Even Gershun has generally utilized only the solenoidal property of the field and has neglected the possibilities associated with the potential and the quasi-potential. In fact, the general feeling seems to be that photic field theory is at best only an academic novelty with no practical applications.

In view of this state of affairs, it seemed advisable to try to formulate photic field theory in a reasonably rigorous manner and to explore its range of application. Although this range is definitely limited—largely because of present limitations in available coordinate systems—the photic field is certainly not a triviality and it should develop as an important tool in the calculation of light.

HISTORY

Because of the cosine relation for incident illumination, there is a formal similarity between light calculation and the projections of a vector. The **D**-vector may be defined as a vector which points in the direction of the net flow of radiant energy and whose magnitude is equal to the net radiant power per unit area crossing a surface that is

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

perpendicular to **D**. Then, for any illuminated surface whose plane does not cut the source, the incident pharosage D_i is

$$D_i = |\mathbf{D}| \cos\theta = \mathbf{D} \cdot \mathbf{N}_1,$$

where θ is the angle between **D** and the inward-drawn normal $\mathbf{N}_1$ to the illuminated surface.

This formal relationship leads naturally to the idea of a pharosage vector or "light vector" **D**. R. Mehmke (1), in a paper published in 1893, says: "Neu hinzu gekommen ist jedoch die Anwendung des Beleuchtungsvektors, mit dessen Einführung ich einen Fortschritt gemach zu haben glaube."

In 1900, R. A. Hermann (2), in the extraordinary ninth chapter of his *Treatise on Geometrical Optics*, used the pharosage vector and the theory of the photic field, even to the introduction of a vector potential. E. P. Hyde (3) in 1907 realized the importance of the solenoidal property,

$$\text{div } \mathbf{D} = 0,$$

of the photic field in free space. Since the field is solenoidal, tubes of flux can be introduced; and many problems involving total flux or flux density can be solved merely by referring to a field map. Hyde worked out several specific cases, including that of the photic field produced by a uniform strip source of infinite length.

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195

In his work as lighting designer, Bassett Jones (4) found a need for equations for **D** produced by surface sources of various shapes. Impressed by the difficulties associated with the classical method of surface integration, Jones in 1910 applied potential theory. He says, ". . . we shall follow Tait in assuming, as a mathematical convenience, a potential function that describes in mathematical terms the character of the light field." Unfortunately, a scalar potential rarely exists in the photic field, so this approach must be used with caution.

In 1924, Fock (5) wrote a masterly treatment of the vector potential in photic field theory. The idea had been presented 24 years earlier by Hermann (2), but Fock does not seem to have been aware of the earlier work. Fock particularly advocated the use of the vector potential in the calculation of total radiant power through a surface, where the usual quadruple integral is reduced to a double integral.

More recently Gershun (6, 7), Gurevich (8), Genkin (9), and others have employed photic field theory. Gershun, in particular, has followed Hyde (3) in utilizing the solenoidal properties of the field and has applied the theory to practical problems of daylight illumination in buildings. Most closely related to the present paper is a treatment by Yamauti (10) in 1932. By use of advanced vector analysis, Yamauti attempted to formulate the basic principles of photic field theory, but he found difficulty in establishing results of satisfactory generality.

CHARACTERISTICS OF THE PHOTIC FIELD

A *photic field* is defined as a region of space in which there is a **D**-vector. The pharosage vector may deal with radiant pharosage D_r (watt m^{-2}), or it may deal with luminous pharosage D_l (lumen m^{-2}) or any other evaluation (erythemal, bactericidal, etc.) of radiation (11). The mathematics are identical in all these cases, so we need not distinguish among them in this paper. The purpose of this and the remaining sections of the paper is to formulate general theorems which will allow photic field theory to be applied to a variety of problems.

Some of these theorems are new, some were developed previously by photometrists, others are familiar from potential theory. It seemed best to collect all of the theorems in one place, even though some are undoubtedly familiar to most readers. Proofs are usually omitted when they are available elsewhere but are sketched when the theorem appears to be new or unfamiliar.

Divergence

Theorem 1

In any region of space in which there is no production or absorption of radiant energy,

$$\operatorname{div} \mathbf{D} = 0.$$

If div **D** = 0, the field is said to be *solenoidal*. Tubes of flux exist, and pharos and average pharosage (11) can be found from a field map by utilizing the fact that the same radiant power is present everywhere in a tube of flux.

Theorem 2

If $p(u')$ represents the radiant power per unit volume, produced or absorbed at a point u' in the field, then

$$\operatorname{div} \mathbf{D} = \pm p,$$

where the plus sign applies to the generation of radiant energy and the negative sign to the dissipation of radiant energy. The quantity p is a constant or a known function of position.

Curl

Theorem 3

The necessary and sufficient condition for the existence of a scalar potential φ is that

$$\operatorname{curl} \mathbf{D} = 0.$$

The scalar potential is then defined by the relation,

$$\mathbf{D} = -\nabla\varphi.$$

If curl **D** = 0, the field is said to be *irrotational*. Equipotential surfaces, everywhere perpendicular to the **D**-vector, then exist and constitute an important concept in potential theory. The magnitude of **D** is everywhere determined by the spacing of the equipotential surfaces.

Theorem 4

The necessary and sufficient condition for the existence of a scalar quasipotential Φ is that

$$\mathbf{D} \cdot \operatorname{curl} \mathbf{D} = 0.$$

If this relation is satisfied, the quasipotential (12) is defined by the equation,

$$\mathbf{D} = -\frac{1}{\zeta(u^i)} \nabla\Phi,$$

where $\zeta(u^i)$ is a scalar integrating factor. Also,

$$\operatorname{curl} (\zeta \mathbf{D}) = 0.$$

As in the irrotational field of Theorem 3, there are surfaces that are everywhere perpendicular to **D**. These are surfaces of constant quasipotential, but their spacing is not sufficient to specify **D**.

Theorem 5

The necessary and sufficient condition for the existence of a vector potential $\boldsymbol{A}$ is that

$$\operatorname{div} \mathbf{D} = 0.$$

The vector potential is then defined by the equation,

$$\mathbf{D} = \operatorname{curl} \boldsymbol{A},$$

and div $\boldsymbol{A}$ is arbitrarily set equal to zero.

Laplace and Poisson Equations

Theorem 6

If div **D** = 0 and curl **D** = 0, then the potential is obtained by solution of Laplace's equation,

$$\nabla^2\varphi = 0.$$

The D-vector is then easily obtained by differentiation:

$$\mathbf{D} = -\nabla\varphi.$$

Proof: From the definition of the potential,

$$\mathbf{D} = -\nabla\varphi.$$

Substitution into div **D** gives

$$\operatorname{div} \mathbf{D} = -\operatorname{div} (\nabla\varphi) = -\nabla^2\varphi = 0.$$

Therefore,

$$\nabla^2\varphi = 0.$$

Theorem 7

If div **D** = 0 and **D**·curl **D** = 0, then the quasipotential is obtained by solution of the equation,

$$\nabla^2\Phi = -\zeta\nabla(1/\zeta)\cdot\nabla\Phi.$$

In the special case that $\nabla(1/\zeta)$ is normal to $\mathbf{D}$,

$$\nabla(1/\zeta)\cdot\nabla\Phi = 0$$

and Laplace's equation must be satisfied:

$$\nabla^2\Phi = 0.$$

When the quasipotential has been determined, $\mathbf{D}$ is obtained from

$$\mathbf{D} = -\frac{1}{\zeta}\nabla\Phi.$$

The integrating factor $\zeta(u^i)$ is specified in such a way that the boundary conditions are satisfied.

Proof: From the definition of the quasipotential,

$$\mathbf{D} = -\frac{1}{\zeta}\nabla\Phi.$$

Thus,

$$\begin{aligned}\operatorname{div}\mathbf{D} &= -\operatorname{div}\left(\frac{1}{\zeta}\nabla\Phi\right)\\ &= -\left[\nabla(1/\zeta)\cdot\nabla\Phi + \frac{1}{\zeta}\operatorname{div}(\nabla\Phi)\right]\\ &= -\left[\nabla(1/\zeta)\cdot\nabla\Phi + \frac{1}{\zeta}\nabla^2\Phi\right] = 0.\end{aligned}$$

Consequently,

$$\nabla^2\Phi = -\zeta\nabla(1/\zeta)\cdot\nabla\Phi.$$

Theorem 8

If curl $\mathbf{D} = 0$ and div $\mathbf{D} = K$, where K is a known function of position, then Poisson's equation applies,

$$\nabla^2\varphi = -K.$$

Theorem 9

If div $\mathbf{D} = K$ and $\mathbf{D}\cdot$curl $\mathbf{D} = 0$, then the quasipotential is obtained by solution of the equation,

$$\nabla^2\Phi = -\zeta[K + \nabla(1/\zeta)\cdot\nabla\Phi].$$

In the special case that $\nabla(1/\zeta)$ is normal to $\mathbf{D}$,

$$\nabla(1/\zeta)\cdot\nabla\Phi = 0$$

and the foregoing equation reduces to Poisson's equation,

$$\nabla^2\Phi = -\zeta K.$$

Theorem 10

If div $\mathbf{D} = 0$ and curl $\mathbf{D} = \mathbf{J}$, where $\mathbf{J}$ is a known vector point function, then $\boldsymbol{A}$ is obtained by solution of the vector Poisson equation,

$$\nabla^2\boldsymbol{A} = -\mathbf{J}.$$

Elementary (Point) Sources

The foregoing theorems (1 to 10) are perfectly general. The medium may be inhomogeneous and non-isotropic; it may be dissipative and diffusing. In the following theorems on elementary sources, however, we make the restriction that the medium be homogeneous and isotropic.

Theorem 11

The necessary and sufficient condition for the existence of a scalar potential φ, in a photic field produced by an elementary source, is that the intensity of the source be uniform in all directions.

Proof: By Theorem 3, the existence of a potential requires that curl $\mathbf{D} = 0$. But the component of curl $\mathbf{D}$ in any direction i may be defined as

$$\text{curl}_i \mathbf{D} = \lim_{\Delta A \to 0} \frac{\oint \mathbf{D} \cdot d\mathbf{s}}{\Delta A},$$

where the line integral is taken around a closed path C in a plane whose normal is in the i-direction. The area ΔA is that enclosed by C.

Consider a set of concentric spheres with the elementary source as center, and a set of radial lines. The line integral about any closed path on a sphere is always zero because $\mathbf{D} \cdot d\mathbf{s} = 0$, but the integral in a plane that includes the source may not be zero. Take a contour consisting of two neighboring arcs and two radial line segments. Evidently, the line integral can be zero only if $\mathbf{D}$ is the same along the two radial sides, which necessitates the same intensity in these two radial directions. Thus curl $\mathbf{D}$ can be zero at every point in the field if and only if the intensity of the source is independent of direction.

Theorem 12

A quasipotential always exists in the photic field produced by a single elementary source, and the surfaces of equal quasipotential are always concentric spheres about the source.

Proof: The $\mathbf{D}$-vector produced by an elementary source must always be radial, since it points in the direction of propagation of radiant energy. Thus $\mathbf{D}$ is always perpendicular to a family of concentric spheres with the source at the center. But if $\mathbf{D}$ is radial, curl $\mathbf{D}$ can have no component in the direction of $\mathbf{D}$ (by definition of curl, Theorem 11). Consequently, $\mathbf{D} \cdot \text{curl } \mathbf{D} = 0$ and by Theorem 4, a quasipotential exists, even though the source is non-uniform.

Theorem 13

The photic field produced in a homogeneous, isotropic medium by a finite number of uniform elementary sources is always irrotational.

Proof: Let the $\mathbf{D}$-vectors at a fixed point P be $\mathbf{D}_1, \mathbf{D}_2, \cdots \mathbf{D}_n$, corresponding to uniform elementary sources $1, 2, \cdots n$. The total pharosage at P is the vector sum,

$$\mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2 + \cdots + \mathbf{D}_n.$$

Also, the component of curl $\mathbf{D}$, perpendicular to the plane of C, is

$$\lim_{\Delta A \to 0} \frac{\oint_C \mathbf{D} \cdot d\mathbf{s}}{\Delta A},$$

where C is a path about P in an arbitrary plane.

Since each elementary source is uniform (I = const.), its curl $\mathbf{D}_i$ is zero according to Theorem 11. Thus taking the same path C that was used in defining the curl of the total field, we have

$$\oint_C \mathbf{D}_1 \cdot d\mathbf{s} = 0,$$
$$\oint_C \mathbf{D}_2 \cdot d\mathbf{s} = 0,$$
$$\cdots\cdots\cdots\cdots$$
$$\oint_C \mathbf{D}_n \cdot d\mathbf{s} = 0.$$

But $\oint_C \mathbf{D} \cdot d\mathbf{s} = \oint_C [\mathbf{D}_1 + \mathbf{D}_2 + \cdots + \mathbf{D}_n] \cdot d\mathbf{s}$
$$= \oint_C \mathbf{D}_1 \cdot d\mathbf{s} + \oint_C \mathbf{D}_2 \cdot d\mathbf{s} + \cdots + \oint_C \mathbf{D}_n \cdot d\mathbf{s} = 0.$$

This is true for any C in any plane through P, so

$$\text{curl } \mathbf{D} = 0,$$

and therefore a potential exists.

Theorem 14

A quasipotential does not ordinarily exist in the photic field produced by a finite number of non-uniform elementary sources.¶

Proof: Proceed as in the proof of Theorem 13. As before,

$$\mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2 + \cdots + \mathbf{D}_n.$$

The individual sources are non-uniform, so curl $\mathbf{D}_i \neq 0$ but curl $(\zeta_i \mathbf{D}_i) = 0$ by Theorem 12. Here ζ_i is a scalar factor which depends on the angle from the source and which is therefore, in general, different for each source, when determined at the fixed point P.

Take C in an arbitrary plane through P and use this same path of integration in all cases. For the resultant field, produced by all the sources, the component of curl in the direction normal to C is

$$\text{curl}_i (\zeta \mathbf{D}) = \lim_{\Delta A \to 0} \frac{\oint_C \zeta \mathbf{D} \cdot d\mathbf{s}}{\Delta A}.$$

For the individual fields,

$$\oint_C \zeta_1 \mathbf{D}_1 \cdot d\mathbf{s} = 0,$$
$$\oint_C \zeta_2 \mathbf{D}_2 \cdot d\mathbf{s} = 0,$$
$$\cdots\cdots\cdots\cdots$$
$$\oint_C \zeta_n \mathbf{D}_n \cdot d\mathbf{s} = 0.$$

But, in general, $[\zeta_1 \mathbf{D}_1 + \zeta_2 \mathbf{D}_2 + \cdots + \zeta_n \mathbf{D}_n]$ is not equal to $\mathbf{D}$ times any scalar factor ζ, since the vector sum generally has a direction which is different from the direction of $\mathbf{D}$. Thus

$$\text{curl } (\zeta \mathbf{D}) \neq 0.$$

Surface Sources

Consider a surface source that is perfectly diffusing and of uniform helios and that coincides with one surface belonging to a triply orthogonal family of surfaces. The source, for example, might be a uniform sphere, or a circular disk, or a long elliptic cylinder, or a prolate spheroid. The medium must be homogeneous and isotropic.

In the interest of easy visualization, we have stated Theorems 15 to 17 in terms of luminous surface sources. As a matter of fact, however, the results are of greater generality. The surfaces of the family need not be physical light sources but may be imaginary surfaces, provided that the helios as measured on the surface is independent of position and angle for one side of the surface.

For instance, consider the photic field beneath an aperture in a horizontal opaque plate. The actual source is a sky of uniform helios, but this source can be replaced by an imaginary plane source at the aperture. In a room lighted through a window, however, such a replacement of the actual source is not permissible since the helios measured at the window is no longer independent of angle, being ordinarily less below the horizon than above the horizon.

Theorem 15

The necessary and sufficient condition for the existence of a scalar potential in a field produced by a perfectly diffusing source of uniform helios, which coincides with a coordinate surface of the triply orthogonal family, is that the distance between two neighboring surfaces of the same kind be everywhere equal.

For instance, consider the photic field produced by a uniform spherical source. Associated with spherical coordinates, we have a triply orthogonal family consisting of spheres, circular cones, and half planes. If the spherical source has uniform helios, the **D**-vector at the surface of the source has constant magnitude and is always radial (Theorem 19). Take a path of integration C in one of the coordinate planes. The path consists of two circular arcs (one on the sphere and one on a neighboring sphere, outside the source) and two radial line segments. The integral

$$\oint_C \mathbf{D}\cdot d\mathbf{s} = \int_1 \mathbf{D}\cdot d\mathbf{s} + \int_2 \mathbf{D}\cdot d\mathbf{s} + \int_3 \mathbf{D}\cdot d\mathbf{s} + \int_4 \mathbf{D}\cdot d\mathbf{s}.$$

The integrals on the arcs 1 and 3 are zero because **D** is everywhere normal to $d\mathbf{s}$. Thus

$$\oint_C \mathbf{D}\cdot d\mathbf{s} = \int_a^{a+\Delta r_2} D\,dr - \int_a^{a+\Delta r_4} D\,dr,$$

both values of D being the same. The only way in which the line integral (and consequently the curl) can be zero is for the distances Δr_2 and Δr_4 to be equal. With spherical surfaces, this requirement is obviously satisfied and the field is irrotational. Evidently, however, the number of possible source shapes is very limited if curl **D** is to be zero.

Theorem 16

The only surface sources (perfectly diffusing and of uniform helios) that produce irrotational photic fields are the plane, the circular cylinder, and the sphere.

The result follows directly from Theorem 15.

Theorem 17

If a luminous surface, perfectly diffusing and of uniform helios, in a perfect medium[4] coincides with a surface of the family, then the other

[4] *A perfect medium* is one that is homogeneous, isotropic, non-dissipative, and non-scattering.

surfaces of the same kind, belonging to the family, are surfaces of constant quasipotential.

The surfaces of constant quasipotential correspond to the equipotential surfaces of the similar electrostatic problem. For example, an isolated, charged elliptic cylinder of infinite length has as equipotentials a family of confocal elliptic cylinders. The corresponding photic field is produced by a perfectly diffusing elliptic cylinder of uniform helios. The equipotentials of the electrostatic problem become the surfaces of constant quasipotential in the photic field. Only in a few cases (Theorem 16) does the integrating factor ζ become unity, in which condition the quasipotential becomes a potential.

Boundary Conditions for Perfectly Diffusing Surfaces in Homogeneous, Isotropic Media

Theorem 18

For any perfectly diffusing source of uniform helios H, the surface having a non-vanishing radius of curvature everywhere, the **D**-vector is perpendicular to the surface throughout and its magnitude is equal to H:

$$|\mathbf{D}| = H.$$

Proof: It is a well-known fact that a perfectly diffusing plane source, of infinite extent and of uniform helios H, produces a **D**-vector whose magnitude is independent of position in the field and is equal to H. The direction of **D** is everywhere perpendicular to the plane of the source.

Now consider another surface, not infinite in extent and not necessarily plane. As a point approaches the luminous surface, the latter appears more and more nearly like an infinite plane source and D becomes more and more nearly equal to H.

Suppose that the luminous surface has a radius of curvature R at the point under consideration. From elementary photometrics,

$$|\mathbf{D}| = H \sin^2 \gamma,$$

where γ is the angle from the axis of the lightcone to its outer extremity. The **D**-vector is on the line from the center of curvature to P. But as P approaches the surface, $\gamma \to \pi/2$ and $|\mathbf{D}| \to H$. This is true of any surface whose radius of curvature is not infinitesimal. Thus an important boundary condition in photic field theory is that, at the surface of the source, **D** is normal to the surface and its magnitude is equal to H.

Theorem 19

For a continuous, perfectly diffusing surface source having a continuous, bounded variation in helios, the **D**-vector is everywhere perpendicular to the surface and its magnitude is equal to the helios of the surface at the point under consideration.

Consider a luminous plane surface extending to $\pm \infty$ in the x-direction and z-direction. The point P is taken at distance y from the luminous surface at $x = 0$. The helios variation on the surface is

$$H(x) = A + B\frac{x}{l}$$

for $-l \leqq x \leqq +l$, $H(x) = A - B$ for $x < -l$, and $H(x) = A + B$ for $x > +l$. From elementary photometrics,

$$\begin{cases} D_y = \dfrac{y^2}{2} \displaystyle\int \frac{H(x)dx}{[x^2 + y^2]^{3/2}}, \\ D_x = \dfrac{y}{2} \displaystyle\int \frac{H(x)xdx}{[x^2 + y^2]^{3/2}}. \end{cases}$$

Thus, taking into account the fact that the x-component of the **D**-vector is the difference of components in the $+x$ and $-x$ directions, we have

$$\begin{cases} D_y = A, \\ D_x = y\left\{ \dfrac{B}{l} \displaystyle\int_0^l \frac{x^2dx}{[x^2 + y^2]^{3/2}} + B \int_l^\infty \frac{xdx}{[x^2 + y^2]^{3/2}} \right\} \end{cases}$$

or

$$D_x = \frac{yB}{l} \ln \left(\frac{l + \sqrt{l^2 + y^2}}{y} \right).$$

If B is bounded and we approach the surface so that $y \ll l$,

$$D_x \to 0.$$

Therefore, the **D**-vector is normal to the surface and has the value of helios at the point in question.

Theorem 20

For any interflection problem in which the surfaces are perfectly diffusing and in which the incident **D**-vector is normal to the surface, a boundary condition is

$$\mathbf{D} = \frac{\mathbf{N}_1}{\rho} [H_0 - (1 - \rho)H],$$

where $\mathbf{N}_1$ is a unit vector, normal to the surface. H is the helios of the surface, including interflections, H_0 in the helios without interflections, and ρ is the reflectance of the surface.

Proof: The **D**-vector at the surface is equal to the vector sum of $\mathbf{D}_r$ from the surface and $\mathbf{D}_i$ incident on the surface, or

$$\mathbf{D} = \mathbf{N}_1(H - D_i).$$

Note that this equation requires that $\mathbf{D}_i$ be normal to the surface. Since the surface is perfectly diffusing, the reflected pharosage vector is always normal to the surface; and by Theorem 19, the vector associated with the self-luminosity of the surface is also normal to the surface.

The true helios H must be the sum of H_0 and that caused by reflected light, so

$$H = H_0 + \rho D_i,$$

or

$$D_i = (H - H_0)/\rho.$$

Substitution gives the boundary condition,

$$\mathbf{D} = \frac{\mathbf{N}_1}{\rho} [H_0 - (1 - \rho)H].$$

General Fields

Theorem 21

For any field that is uniform in the z-direction and extends to $\pm\infty$ in this direction, a quasipotential always exists.

Proof: Consider any point P, outside of the source. A **D**-vector is associated with this point, and the vector is in the xy-plane. Pass a plane through P, oriented so that its normal is in the direction of **D**. Obtain the line integral of **D** on a small closed path C about P, C being in the plane that is normal to **D**.

In general, **D** will not be perpendicular to the contour C but will have a small component along the curve, but since everything is the same at $z + \Delta z$ and at $z - \Delta z$, these components of **D** will cancel and

$$\oint_C \mathbf{D}\cdot d\mathbf{s} = 0.$$

Thus *there is no component of curl* **D** *in the direction of* **D**. Therefore,

$$\mathbf{D}\cdot\text{curl}\,\mathbf{D} = 0$$

and a quasipotential exists.

An alternative proof utilizes the expression for curl **D** in general cylindrical coordinates. Since **D** has no component in the z-direction,

$$\mathbf{D} = \mathbf{a}_1 D_1(u^1, u^2) + \mathbf{a}_2 D_2(u^1, u^2).$$

Also,

$$\text{curl}\,\mathbf{D} = \begin{vmatrix} \sqrt{\frac{g_{11}}{g}}\,\mathbf{a}_1 & \sqrt{\frac{g_{22}}{g}}\,\mathbf{a}_2 & \sqrt{\frac{1}{g}}\,\mathbf{a}_3 \\ \frac{\partial}{\partial u^1} & \frac{\partial}{\partial u^2} & \frac{\partial}{\partial u^3} \\ \sqrt{g_{11}}\,D_1 & \sqrt{g_{22}}\,D_2 & 0 \end{vmatrix}.$$

In a cylindrical coordinate system, the metric coefficients g_{11} and g_{22} are at most functions of u^1 and u^2 only, so

$$\text{curl}\,\mathbf{D} = \frac{1}{\sqrt{g}}\,\mathbf{a}_3\left[\frac{\partial}{\partial u^1}\left(\sqrt{g_{22}}\,D_2\right) - \frac{\partial}{\partial u^2}\left(\sqrt{g_{11}}\,D_1\right)\right].$$

Therefore curl **D** is in the z-direction. Since curl **D** has no component in the direction of **D**,

$$\mathbf{D}\cdot\text{curl}\,\mathbf{D} = 0.$$

Theorem 22

A quasipotential exists in any field that has axial symmetry.

Consider a plane that is determined by the arbitrary point P and the axis of symmetry. The **D**-vector lies in this plane because of symmetry. Take a closed path C, about P and in the plane whose normal is in the direction of **D**. In general, **D** will not be perpendicular to $d\mathbf{s}$ on the contour. But it is evident that the components of **D** along C will cancel because of symmetry. Thus

$$\oint_C \mathbf{D}\cdot d\mathbf{s} = 0$$

and the component of curl **D** in the direction of **D** is zero. Consequently, a quasipotential exists.

An alternative proof employs the expression for curl **D** in a coordinate system with axial symmetry. The proof proceeds as in Theorem 21.

Theorems 21 and 22 are not restricted to surface sources of uniform helios. The theorems are general and apply to any source—point, line, surface, or volume.

The Vector Potential in a Perfect Medium

Theorem 23

For a solenoidal photic field in a perfect medium and a receiving surface S whose boundary C can be seen from every point of a source, the pharos from the source to S can be expressed as the line integral of A about C:

$$F = \oint_C A \cdot d\mathbf{s}.$$

Proof: The total pharos to S is

$$F = \int_S \mathbf{D} \cdot d\mathbf{A}.$$

But $\mathbf{D} = \text{curl } A$, so

$$F = \int_S \text{curl } A \cdot d\mathbf{A}.$$

Stokes' theorem states that

$$\int_S \text{curl } A \cdot d\mathbf{A} = \oint_C A \cdot d\mathbf{s},$$

so the surface integral can be replaced by a line integral about the boundary of the receiving surface, or

$$F = \oint_C A \cdot d\mathbf{s}.$$

Theorem 24

The pharosage produced by a perfectly diffusing surface source of uniform helios H is proportional to the line integral of $d\boldsymbol{\alpha}$ along the boundary of the source:

$$\mathbf{D} = \frac{H}{2\pi} \oint d\boldsymbol{\alpha},$$

where $d\boldsymbol{\alpha}$ is a vector whose magnitude is equal to the angle subtended by $d\mathbf{s}$ at P and whose direction is normal to the plane of $d\mathbf{s}$ and $\mathbf{r}$.

Proof: The pharosage produced at P, by an element dA of perfectly diffusing surface having helios H, is

$$|d\mathbf{D}| = \frac{H dA}{r^2} \cos(\mathbf{r}_1, d\mathbf{A}),$$

where r is the distance from dA to P. Thus, for a surface source S of uniform helios H,

$$\mathbf{D} = \frac{\mathbf{r}_1 H}{r} \int_S \frac{\mathbf{r}_1 \cdot d\mathbf{A}}{r^2}.$$

Here $\mathbf{r}_1$ is a unit vector directed from $d\mathbf{A}$ toward P.

Multiply both sides by an arbitrary vector $\mathbf{N}_1$:

$$\begin{aligned}\mathbf{D}\cdot\mathbf{N}_1 &= (\mathbf{r}_1\cdot\mathbf{N}_1)\frac{H}{\pi}\int_S\left(\frac{\mathbf{r}_1}{r^2}\right)\cdot d\mathbf{A}\\ &= \frac{H}{\pi}\int_S\left[\frac{\mathbf{r}_1}{r^2}(\mathbf{r}_1\cdot\mathbf{N}_1)\right]\cdot d\mathbf{A}\\ &= \frac{H}{2\pi}\int_S \operatorname{curl}\left(\frac{\mathbf{r}_1}{r}\times\mathbf{N}_1\right)\cdot d\mathbf{A}.\end{aligned}$$

By Stokes' theorem,

$$\int_S \operatorname{curl}\mathbf{v}\cdot d\mathbf{A} = \oint_C \mathbf{v}\cdot d\mathbf{s}.$$

So

$$\mathbf{D}\cdot\mathbf{N}_1 = \frac{H}{2\pi}\oint\frac{\mathbf{r}_1}{r}\times\mathbf{N}_1\cdot d\mathbf{s} = \left[\frac{H}{2\pi}\oint d\mathbf{s}\times\frac{\mathbf{r}_1}{r}\right]\cdot\mathbf{N}_1$$

by cyclic permutation of the elements in the scalar triple product.

Thus,

$$\mathbf{D} = \frac{H}{2\pi}\oint d\mathbf{s}\times\frac{\mathbf{r}_1}{r} = \frac{H}{2\pi}\oint d\alpha.$$

If the source has a polygonal boundary, the above equation reduces to

$$\mathbf{D} = \frac{H}{2\pi}\sum_{i=1}^{n}\alpha_i,$$

where $|\alpha_i|$ is the angle subtended at point P by the ith side of the source ($i = 1, 2, \cdots n$). This theorem is discussed in Moon, *Scientific Basis of Illuminating Engineering*, p. 314 and is proved by Fock (5), Gershun and Gurevich (8), and Yamauti (10).

Theorem 25

The vector potential, produced by a perfectly diffusing surface source of uniform helios, having a boundary that is invariant with respect to the position of P, is

$$A = -\frac{H}{2\pi}\oint_C \ln r\, d\mathbf{s}$$

where $d\mathbf{s}$ is an element of the path C and r is the distance from $d\mathbf{s}$ to P.

Proof: From the proof of Theorem 24,

$$\mathbf{D} = \frac{H}{2\pi}\oint d\mathbf{s}\times\frac{\mathbf{r}_1}{r}.$$

But

$$\frac{\mathbf{r}_1}{r} = \nabla(\ln r),$$

so

$$\mathbf{D} = -\frac{H}{2\pi}\oint\nabla(\ln r)\times d\mathbf{s}.$$

From (13)

$$\begin{aligned}\nabla(\ln r)\times d\mathbf{s} &= \operatorname{curl}(\ln r\, d\mathbf{s}) - \ln r\operatorname{curl}(d\mathbf{s})\\ &= \operatorname{curl}(\ln r\, d\mathbf{s}).\end{aligned}$$

Thus

$$\mathbf{D} = -\frac{H}{2\pi} \oint \operatorname{curl} (\ln r \, d\mathbf{s})$$

and if the entire curve C is visible from P and its vicinity,

$$\mathbf{D} = -\operatorname{curl} \left[\frac{H}{2\pi} \oint \ln r \, d\mathbf{s} \right].$$

But $\mathbf{D} = \operatorname{curl} \mathbf{A}$, so the vector potential is

$$\mathbf{A} = -\frac{H}{2\pi} \oint \ln r \, d\mathbf{s}.$$

AN EXAMPLE

As an application of photic field theory, consider the field produced in a non-dissipative medium by an oblate spheroidal source having a perfectly diffusing surface of uniform helios. Oblate spheroidal coordinates (η, θ, ψ) will be used. By Theorem 22, a quasipotential exists because of axial symmetry. By Theorem 1,

$$\operatorname{div} \mathbf{D} = 0;$$

and according to Theorem 7, it is reasonable to assume that Laplace's equation applies:

$$\nabla^2 \Phi = 0$$

and

$$\mathbf{D} = -\frac{1}{\zeta} \nabla \Phi.$$

The pharosage vector $\mathbf{D}$ is constant in magnitude over the source and is normal to the surface (Theorem 18), so boundary conditions are

$$\begin{cases} \text{When } \eta = \eta_0, & D_\eta = H \text{ and } D_\theta = 0; \\ \text{When } \eta \to \infty, & D_\eta = 0 \text{ and } D_\theta = 0. \end{cases}$$

Also, Laplace's equation (14) in oblate spheroidal coordinates is

$$\frac{d^2\Phi}{d\eta^2} + \tanh \eta \frac{d\Phi}{d\eta} = 0, \tag{1}$$

whose general solution is

$$\Phi = A + B \tan^{-1} (\sinh \eta). \tag{2}$$

The gradient (14) is

$$\nabla \Phi = \frac{\mathbf{a}_\eta}{c \sqrt{\cosh^2 \eta - \sin^2 \theta}} \frac{d\Phi}{d\eta},$$

so

$$\mathbf{D} = -\frac{1}{\zeta}\frac{\mathbf{a}_\eta B}{c\cosh\eta\sqrt{\cosh^2\eta - \sin^2\theta}}. \tag{3}$$

To fit the boundary conditions, we make

$$\zeta = \frac{1}{c\sqrt{\cosh^2\eta - \sin^2\theta}}. \tag{4}$$

Then at $\eta = \eta_0$,

$$H = -\frac{B\sqrt{\cosh^2\eta_0 - \sin^2\theta}}{\cosh\eta_0\sqrt{\cosh^2\eta_0 - \sin^2\theta}}$$

or

$$B = -H\cosh\eta_0. \tag{5}$$

The zero of quasipotential (like the zero potential) may be chosen arbitrarily. If $\Phi = 0$ when $\eta = \eta_0$, then

$$A = H\cosh\eta_0\tan^{-1}(\sinh\eta_0)$$

and the solution of the problem is

$$\begin{cases} \Phi = H\cosh\eta_0\left[\tan^{-1}(\sinh\eta_0) - \tan^{-1}(\sinh\eta)\right], \\ \mathbf{D} = \mathbf{a}_\eta H\dfrac{\cosh\eta_0}{\cosh\eta}\sqrt{\dfrac{\cosh^2\eta_0 - \sin^2\theta}{\cosh^2\eta - \sin^2\theta}}. \end{cases} \tag{6}$$

Since $\zeta = \zeta(\theta)$ and $\Phi = \Phi(\eta)$, the auxiliary condition is satisfied and the assumption of Laplace's equation is valid.

The results are most simply expressed in terms of η and θ. If desired, however, they can be expressed in terms of rectangular coordinates, using the relations

$$\begin{cases} x = c\cosh\eta\sin\theta\cos\psi, \\ y = c\cosh\eta\sin\theta\sin\psi, \\ z = c\sinh\eta\cos\theta. \end{cases} \tag{7}$$

For the special case of a circular disk source, $\eta_0 = 0$ and Eq. 6 becomes

$$\mathbf{D} = \mathbf{a}_\eta H\frac{\cos\theta}{\cosh\eta\sqrt{\cosh^2\eta - \sin^2\theta}}. \tag{6a}$$

Equation 6a applies to any point in the field. On the axis, $\theta = 0$ and Eq. 6a becomes

$$D = \frac{H}{\cosh^2\eta} = \frac{H}{1 + \sinh^2\eta}.$$

But on the axis,

$$z = c \sinh \eta,$$

and substitution yields the customary formula for pharosage on the axis of a uniform disk source:

$$D = H \sin^2 \gamma, \tag{6b}$$

where $\gamma = \tan^{-1}(c/z)$, the half angle subtended by the disk of radius c at the point P.

Another result that can be obtained from photic field theory is the total radiant power (pharos) to a surface or through an aperture. Suppose that we want an equation for the pharos through a circular aperture of radius b, at distance l from a circular disk source of radius c and helios H. The aperture is concentric with the disk and parallel to it.

Since the field is solenoidal, the pharos through the aperture is the same as the pharos from the portion of the source that lies within the largest tube of flux that can pass through the aperture. From Eq. 7,

$$l/c = \sinh \eta \cos \theta,$$
$$b/c = \cosh \eta \sin \theta,$$

or

$$c^2 = \frac{b^2}{\sin^2 \theta} - \frac{l^2}{\cos^2 \theta}.$$

Solving for $\sin^2 \theta$, we obtain

$$\sin^2 \theta = \frac{1}{2c^2}\left[(b^2 + c^2 + l^2) \pm \sqrt{(b^2 + c^2 + l^2)^2 - 4b^2c^2}\right]. \tag{8}$$

This tube of flux, characterized by $\theta = \text{const.}$, cuts the source at $\eta = 0$, $x = c \sin \theta$. The pharos leaving the source within this tube is

$$F = \pi x^2 H = \pi H c^2 \sin^2 \theta.$$

Substitution of Eq. 8 gives

$$F = \frac{\pi H}{2}\left[(b^2 + c^2 + l^2) \pm \sqrt{(b^2 + c^2 + l^2)^2 - 4b^2c^2}\right]. \tag{9}$$

The equations obtained in this section can be derived in the classical manner by surface integration. Note, however, that photic field theory gives the results without any integration or other arduous manipulation. Thus, when a coordinate system can be found to fit the problem under consideration, there is a distinct advantage in using photic field theory.

SUMMARY

The application of field theory to the calculation of amounts of light has been advocated by a number of investigators. The general feeling, however, seems to be that photic field theory is applicable in so few cases that it is hardly worthy of consideration. A new study of the subject shows that it is of much wider applicability than has been customarily believed. Particularly by the use of the quasipotential, one is able to obtain results very simply that would require troublesome integrations by the classical method.

In a few cases, such as the uniform point source, the uniform sphere, and the uniform circular cylinder, the photic field is mathematically identical with the electrostatic field. In these cases, classical potential theory can be applied directly. With most photic fields, however, a scalar potential does not exist and methods must be modified. In all problems having axial symmetry, as well as in other important cases, a quasipotential exists and the procedure is similar to that for a potential field. In still other cases, neither a potential nor a quasipotential exists and one must resort either to a vector potential or to the common photometric methods of calculation.

In the foregoing paper, we have attempted to formulate the subject in an exact manner. Theorems have been stated regarding the conditions for the existence of a potential, a quasipotential, and a vector potential in the photic field. Methods of solution are outlined and boundary conditions are formulated. Finally, an example is worked out.

Of particular importance are the new Theorems Nos. 7 and 9 on the quasipotential and Nos. 18 and 19 on boundary conditions. Theorem 16 lists the surface sources that produce irrotational fields, while Theorems 21 and 22 indicate general classes of surfaces for which a quasipotential exists.

The practical limitation of photic field applications is primarily one of finding a coordinate system. If the luminous surface corresponds to one of the familiar coordinate surfaces (planes, circular cylinders, elliptic cylinders, hyperbolic cylinders, parabolic cylinders, oblate spheroids, paraboloids, etc.), there is ordinarily no difficulty in solving the photic problem with uniform sources. With other problems, difficulty may be encountered, just as in the corresponding electrostatic problem.

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ELEMENTARY PROBABILITY OF EFFICIENT ABSORPTION OF A QUANTUM PROPORTIONAL TO THE VOLUME OF A GRAIN

BY

LUDWIK SILBERSTEIN[1]

We have assumed in all previous investigations that the elementary probability of an efficient absorption by a solitary grain of a single quantum striking a plate (of unit area) is proportional to the projected area a of the grain,

$$p = \epsilon a. \tag{1}$$

This assumption is, tacitly, based on the belief that the process of rendering the grain developable (whether, for example, by producing a free electron within the crystalline grain or by any other mechanism) is seated in the *surface layer* of the grain which has been likened to a flat target. It implies that the developer, after exposure, attacks exclusively, or nearly so, only such portions of the silver halide modified by the light quantum which lie at or very near the surface of the grain target, briefly, that the action of the developer is, originally at least, but skin deep. Such a view would harmonize, for instance, with the so-called "silver electrode theory" of Gurney and Mott.[2]

Although the implications of the assumption (1) have been seen to fit the experimental facts rather closely,[3] it may not be out of place to consider here the implications of an alternative assumption, namely, that of proportionality of the elementary probability p to the volume (v) of the silver halide grain, whatever its shape, for example, quasi-spherical or target-like. In symbols,

$$p = \zeta v, \tag{2}$$

where ζ (replacing our previous ϵ) is some numerical factor, presumably a small fraction.

The probability that the grain (a *single* grain present on the plate) shall efficiently absorb at least r quanta out of a total of n quanta

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[1] Deceased; formerly, Research Consultant, Rochester, N. Y.

[2] R. W. Gurney and N. F. Mott, *Proc. Roy. Soc. 1938*, Vol. 164A, p. 151. A short account of this theory is given on pp. 184 *et seq.* of the "Theory of the Photographic Process" by C. E. K. Mees, 1942. The formation of the latent image was described as a Becquerel cell by A. P. H. Trivelli in the JOUR. FRANKLIN INST., Vol. 205, p. 111 (1928).

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thrown upon the plate or film per unit area is, as before,

$$P_r = 1 - e^{-pn} \sum_{\kappa=1}^{r-1} \frac{(pn)^\kappa}{\kappa!},$$

and thus, by (2),

$$P_r = 1 - e^{-\zeta vn} \sum_{\kappa=1}^{r-1} \frac{(\zeta vn)^\kappa}{\kappa!}. \tag{3}$$

For $r = 1$,

$$P_1 = 1 - e^{-\zeta vn},$$

and so on.

If there are N grains spread over our plate (of unit area), all of the same volume v, then the probable number of grains rendered developable by the exposure n will be, under the r-quantic hypothesis,

$$k = NP_r = N - Ne^{-\zeta vn} \sum_{\kappa=1}^{r-1} \frac{(\zeta vn)^\kappa}{\kappa!}. \tag{4}$$

Such then, would be our modified microphotographic formula. As regards the *macroscopic density* D produced by an exposure n, it will manifestly depend on the distribution of "sizes," projected areas a, among the N grains, all of the same volume v. If the number of grains of size a to $a + da$ is $N \cdot f(a)da$, where $f(a)$ is some function satisfying the obvious condition

$$\int_0^\infty f(a)da = 1,$$

then

$$D = MN \int_0^\infty af(a)P_r da, \qquad M = \log e.$$

For $n \to \infty$, $P_r \to 1$ and D becomes the limiting density, D_m, so that

$$D_m = MN \int_0^\infty af(a)da = MN\bar{a},$$

where $\bar{a}$ is the mean grain size. Thus

$$\frac{D}{D_m} = \frac{1}{\bar{a}} \int_0^\infty af(a)P_r da$$

and, therefore, by (3)

$$\frac{D}{D_m} = 1 - \frac{1}{\bar{a}} \int_0^\infty af(a)e^{-\zeta vn} \sum_{\kappa=1}^{r-1} \frac{(\zeta vn)^\kappa}{\kappa!} da.$$

But v is common to all N grains, independent of a. And if such also is the case of the coefficient, ζ, we have simply

$$\frac{D}{D_m} = 1 - e^{-\zeta vn} \sum_{\kappa=1}^{r-1} \frac{(\zeta vn)^\kappa}{\kappa!} \tag{5}$$

For example, for $r = 1$ or 2,

$$\frac{D}{D_m} = 1 - e^{-\zeta vn} \text{ and } \frac{D}{D_m} = 1 - e^{-\zeta vn}(1 + \zeta vn),$$

respectively. In fine, our previous argument $y = \epsilon an$ is here replaced by ζvn.

If the N grains are all spherical, of radius R say, then $v = \frac{4\pi}{3} R^3$ being equal for all, so is R and therefore also the "size," $a = \pi R^2$.

If, as in many actual cases, all the grains resemble *flat targets*, say parallel to the base, of area a and of thickness b, then the volume

$$v = ab$$

is, by assumption, common to all N grains, but their sizes a, and therefore also their thicknesses b may be different. If, as in the general case, there are $Nf(a)da$ grains of size a to $a + da$, the formula (5) is still in rigor, that is to say,

$$\frac{D}{D_m} = 1 - e^{-\zeta abn} \sum_{\kappa=1}^{r-1} \frac{(\zeta abn)^\kappa}{\kappa!}. \tag{5a}$$

Consider now the case, of practical interest, in which the N-target-like grains are all of *the same size* a, but of *different thicknesses* b (and thus of different volumes v). Let $Ng(b)db$ be the number of those grains whose thickness ranges from b to $b + db$. Then

$$\frac{k}{N} = \int_0^\infty g(b) \left\{ 1 - e^{-\zeta anb} \sum_{\kappa=1}^{r-1} \frac{(\zeta anb)^\kappa}{\kappa!} \right\} db,$$

or, since again

$$\int_0^\infty g(b)db = 1,$$

$$\frac{k}{N} = 1 - \sum_{\kappa=1}^{r-1} \int_0^\infty g(b) e^{-\zeta anb} \frac{(\zeta anb)^\kappa}{\kappa!} db \tag{6}$$

and if we write $x = \zeta anb$,

$$\frac{k}{N} = 1 - \sum_{\kappa=1}^{r-1} \frac{1}{(\zeta an)^{\kappa+1}} \int_0^\infty g(x') e^{-x} \frac{x^\kappa}{\kappa!} dx, \tag{6a}$$

where $x' = \frac{x}{\zeta an}$.

To illustrate this general formula, let us consider the case of $r = 1$ (*one*-quantic hypothesis) and of a simple *exponential* distribution of thicknesses b among the N grains (all of size a). If β be the *mean* thickness, then, in view of the condition $\int_0^\infty g(b)db = 1$, the function $g(b)$ is completely determined, *viz.*,

$$g(b) = \frac{1}{\beta} e^{-b/\beta}. \tag{7}$$

Thus the formula (6) becomes

$$\frac{k}{N} = 1 - \frac{1}{\beta}\int_0^\infty e^{-b/\beta - \zeta a n b} db.$$

The integral is equal to

$$\left(\frac{1}{\beta} + \zeta a n\right)^{-1} \int_0^\infty e^{-u} du = \left(\zeta a n + \frac{1}{\beta}\right)^{-1}.$$

Thus, ultimately,

$$\frac{k}{N} = \frac{\zeta a \beta n}{1 + \zeta a \beta n}$$

or, since $a\beta = \bar{v}$ is the *mean volume* of the grains,

$$\frac{k}{N} = \frac{\zeta \bar{v} n}{1 + \zeta \bar{v} n}. \tag{8}$$

The exposure E, say in ergs/cm.^{2}, is $n \cdot \frac{hc}{\lambda}$, so that

$$\zeta \bar{v} n = \frac{\zeta \bar{v} \lambda}{hc} E. \tag{9}$$

Our original formula,[4] based on the surface-hypothesis or $p = \epsilon a$, was

$$\frac{k_s}{N} = 1 - e^{-\epsilon a n}, \tag{8$_s$}$$

where the subscript s is to remind us of "surface" (as against "volume"). The exponential function of n is now, in (8), replaced by a simple algebraic function of n.

If we put $y = \epsilon a n$, then

$$\zeta \bar{v} n = \kappa y,$$

where

$$\kappa = \frac{\epsilon a}{\zeta \bar{v}} = \frac{\epsilon}{\beta \zeta}, \tag{10}$$

and (8) and (8$_s$) become

$$k = N \frac{\kappa y}{1 + \kappa y} \tag{8'}$$

and

$$k_s = N(1 - e^{-y}). \tag{8'$_s$}$$

Both numbers tend asymptotically to N, for $y \to \infty$, while, initially, for $y \to 0$,

$$\frac{k}{k_s} = \kappa.$$

The comparison of the two curves (8') and 8'$_s$), for $\kappa = 1$, is shown in Fig. 1. They differ from each other considerably.

[4] L. Silberstein and A. P. H. Trivelli, *Phil. Mag.*, Vol. 44, p. 956 (1922).

This circumstance, then, may afford an experimental method discriminating between the surface-hypothesis ($p = \epsilon a$) and the volume-hypothesis ($p = \zeta v$), if grains of the same size-class (a) do actually have a perceptibly wide range of thicknesses.

The curve k, log y corresponding to any other value of κ will, of course, follow from the $\kappa = 1$ curve by a rigid shift through log κ along the log-exposure axis.

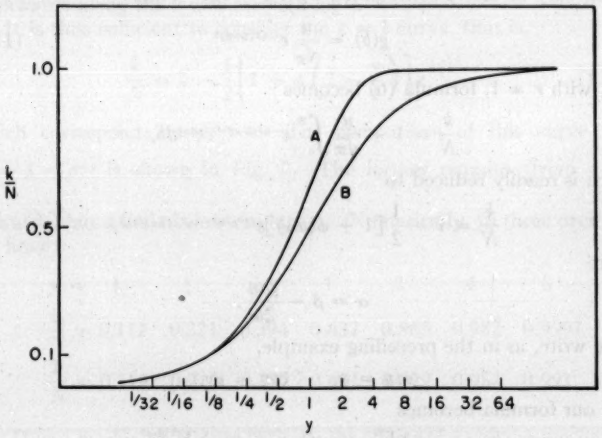


FIG. 1.

Retaining the *one*-quantic hypothesis ($r = 1$), let us still consider a "normal" or Gaussian distribution of thicknesses b among the N grains, all of size a, say

$$g(b) = Ae^{-u^2(b-b_m)^2}. \tag{11}$$

Since $\int_0^\infty g(b)db = 1$, we have necessarily the relation

$$A = \frac{2u}{\sqrt{\pi}[1 + \phi(ub_m)]}, \tag{11.1}$$

where ϕ is the error function. The mean thickness $\beta = \int_0^\infty bg(b)db$ is

$$\beta = \frac{A}{u}\int_{-ub_m}^{\infty}\left(b_m + \frac{x}{u}\right)e^{-x^2}dx = \frac{A}{u}\left\{b_m\frac{\sqrt{\pi}}{2}[1 + \phi(ub_m)] + \frac{e^{-u^2b_m^2}}{2u}\right\}$$

or, by (11.1),

$$\beta = b_m + \frac{e^{-u^2b_m^2}}{\sqrt{\pi}u[1 + \phi(ub_m)]}. \tag{11.2}$$

If the $g(b)$ curve is to be rather slender (standard deviation small compared with mean thickness), ub_m must be a considerable number, say of the order of two or three units, and thus $\phi(ub_m) \doteqdot 1$ and the last two formulas reduce to

$$k = \frac{u}{\sqrt{\pi}} \text{ and } \beta = b_m + \frac{e^{-u^2 b_m^2}}{2\sqrt{\pi}u},$$

which may be safely confounded with $\beta = b_m$. Thus

$$g(b) = \frac{u}{\sqrt{\pi}} e^{-u^2(b-\beta)^2} \tag{11.3}$$

and, with $r = 1$, formula (6) becomes

$$\frac{k}{N} = 1 - \frac{u}{\sqrt{\pi}} \int_0^\infty e^{-u^2(b-\beta)^2 - \zeta a n b} db,$$

which is readily reduced to

$$\frac{k}{N} = 1 - \frac{1}{2}[1 + \phi(u\alpha)] e^{(\zeta^2 a^2 n^2 - 4u^2 \cdot \zeta \beta n)/4u^4},$$

where

$$\alpha = \beta - \frac{\zeta a n}{2u^2}.$$

If we write, as in the preceding example,

$$\epsilon a n = y, \quad \zeta \beta n = \kappa y,$$

then our formula becomes

$$\frac{k}{N} = 1 - \frac{1}{2}\left[1 + \phi\left(u\beta - \frac{\kappa y}{2u\beta}\right)\right] e^{\frac{\kappa^2 y^2}{4u^4\beta^2} - \kappa y}. \tag{12}$$

This is already based on the assumption that $u\beta \doteqdot ub_m$ amounts to a few units. To fix the ideas, let us take

$$u\beta = 2 \qquad (\phi(2) = 0.995 \doteqdot 1).$$

Then

$$\frac{k}{N} = 1 - \frac{1}{2}\left[1 + \phi\left(2 - \frac{\kappa y}{4}\right)\right] e^{\left(\frac{\kappa y}{4}\right)^2 - \kappa y}, \tag{12a}$$

where $\kappa = \dfrac{\beta\zeta}{\epsilon}$ is a pure number. This formula holds for a distribution of thicknesses (11.3) which now becomes

$$g(b) = \frac{2}{\sqrt{\pi}\beta} e^{-\frac{2}{\beta^2}(b-\beta)^2}. \tag{13a}$$

Let us again compare the formula (12a) based on the volume-hypothesis with the formula for the same emulsion treated by the surface-hypothesis,

$$\frac{k_s}{N} = 1 - e^{-y}, \qquad y = ena.$$

Notice that for any value of the parameter κ as compared with $\kappa = 1$ we have, by (12a),

$$k_\kappa(y) = k_1(\kappa y)$$

or, if $x = \log y$,

$$k_\kappa(x) = k_1(x + \log \kappa).$$

In fine, the k_κ-curve is obtained from the k_1-curve shifting it *rigidly* backwards along the x-axis through $\log \kappa$.

It is thus sufficient to consider the $\kappa = 1$ curve, that is,

$$\frac{k}{N} = 1 - \frac{1}{2}\left[1 + \phi\left(2 - \frac{y}{4}\right)\right] e^{\left(\frac{y}{4}\right)^2 - y}, \qquad (12a.1)$$

which corresponds to $\beta\zeta = \epsilon$. The comparison of this curve with $\frac{k_s}{N} = 1 - e^{-y}$ is shown in Fig. 2. The former remains (from $y = \frac{1}{2}$ onwards) but a little below the latter. Numerically, to three decimals, we have:

$y =$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$\frac{k_s}{N} =$	0.112	0.221	0.394	0.632	0.865	0.982	0.9997
$\frac{k}{N} =$	0.119	0.212	0.387	0.601	0.829	0.954	0.991

Thus a marked difference between the effects of a volume-mechanism and a surface-mechanism ($p = \zeta v$, $p = \epsilon a$) can be expected only provided that ϵ differs considerably from $\beta\zeta$ or $\delta\zeta$, when the two curves drawn in Fig. 2 are shifted well apart.

This, it will be recalled, holds for a Gaussian distribution of thickness among the grains, with a rather small "spread." When the distribution is broad and can be approximated by an exponential $g(b)$, the formula (8′) holds which, even for $\kappa = 1$, gives a curve differing considerably from $\frac{k_s}{N} = 1 - e^{-y}$, as in Fig. 1. Numerically, we have at

$y =$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
$(1 + y) =$	0.200	0.333	0.500	0.667	0.800
$e^{-y} =$	0.221	0.394	0.632	0.862	0.982
$\Delta =$	0.021	0.061	0.132	0.198	0.182

The greatest difference $\Delta(y) = 1 - e^{-y} - \frac{y}{1 + y}$ is attained when $e^y = (1 + y)^2$ or

$$0.4343y - 2 \log (1 + y) = 0,$$

which, to three decimals, is satisfied by $y = y_m = 2.514$, so that the maximum is $\Delta(y_m) = 0.204$, not much greater than $\Delta(2) = 0.198$.

The small deviation of k_1, for $u\beta = 2$, from $k_s = N(1 - e^{-y})$ need not surprise us. In fact, returning to formula (12) we have, for $\kappa = 1$,

$$\frac{k_1}{N} = 1 - \frac{1}{2}\left[1 + \phi\left(u\beta - \frac{y}{2u\beta}\right)\right] e^{\left(\frac{y}{2u\beta}\right)^2 - y} \tag{12.1}$$

which holds for $g(b) = \frac{2}{\sqrt{\pi}\beta} e^{-u^2(b-\beta)^2}$. Now, if $u\beta \to \infty$, the thicknesses of all grains tend to equality which ought to obliterate all differences

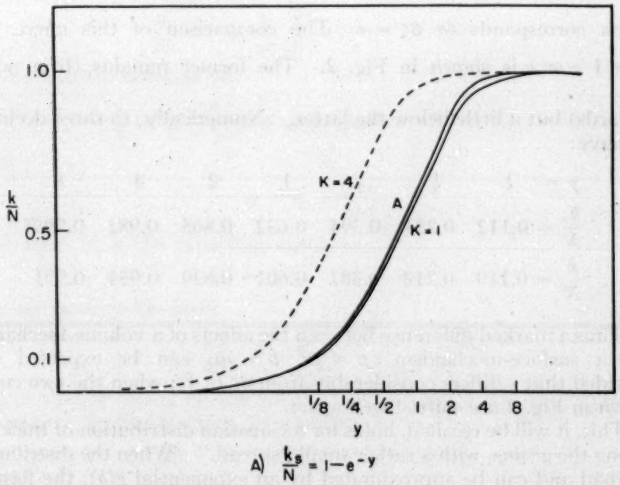


FIG. 2.

between the surface- and the volume-hypotheses. In fact, with ever-increasing $u\beta$, the term $\phi\left(u\beta - \frac{y}{2u\beta}\right)$ tends to unity, for all finite exposures, so that $\frac{k_1}{N}$ tends to $1 - e^{-y}$, for all exposures y small in comparison with $u\beta$.

The analysis of these two cases, *viz.*, of an exponential and a Gaussian distribution of thicknesses among the grains, ought to suffice for a first orientation as to a possible experimental discrimination between our original assumption, $p = \epsilon a$, and its alternative $p = \zeta v$.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

AN AIRBORNE, INFRARED SPECTROGRAPH

Engineers of the Electrical Engineering Division have designed and built an infrared spectrograph for use in an aircraft. In this instrument modern electronic circuits are combined with an optical system originated in the Laboratories by Agnew and his co-workers.[1] The spectrograph is designed to study rapidly varying infrared phenomena at high altitudes. Its detectors are therefore sensitive electrical devices, and the output signals are amplified by high-gain a-c. amplifiers. Cathode-ray oscilloscopes with rapid response display the output signals before the recording cameras. For every second that the instrument is operated, it records on motion picture film twelve complete analyses of intensities in the radiation spectrum of an infrared source. With a single setting, each analysis contains an index of total radiant energy in the band from 1 to 25 microns, and ten sample intensities at specified wavelengths within that region.

In order for an a-c. detector to measure radiation from a steady source, the energy must be modulated or "chopped." Therefore, a rotating mirror is placed ahead of the entrance slit to "chop" the total radiation. The chopper also introduces a standard of radiation contained in the instrument and provides a comparison with the observed infrared source. The chopper, rotating at 12 cycles per second, standardizes first the total radiation channel, then the spectral channels.

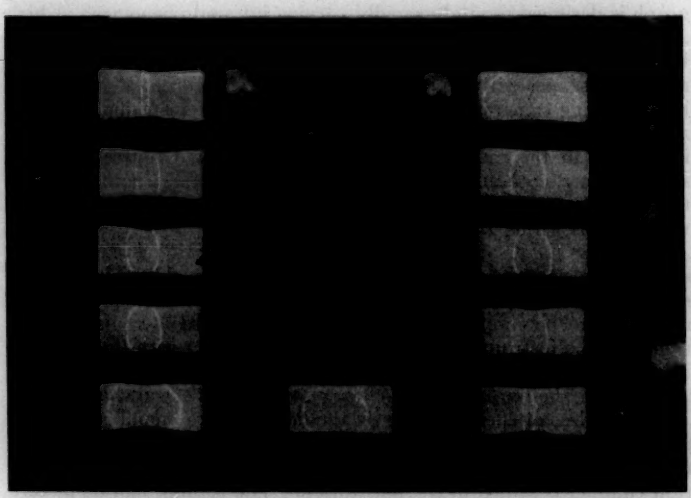
The pulses of energy falling on the detectors are amplified and displayed on narrow, rectangular, cathode-ray tubes. The display theater containing tubes for ten spectral channels and the total-radiation channel is shown in the illustration. The 35-mm. motion-picture camera which photographs these tubes is synchronized with the radiation chopper and the electronic sweep circuits. To supplement the spectral information, various flight and atmospheric instruments which have a bearing upon the radiation records are photographed. The recording facilities include, therefore, a dark theater and a light theater.

When looking at a source of radiation, the operator sees not only the radiant object, but he also observes in a telescope the faint superimposed image of a cathode-ray tube presenting the total energy signal. To further assist in aiming, a varying-pitch audio tone is developed and applied to one of the operator's headphones, giving him another indication of tracking performance.

[1] S. T. Agnew, R. G. Franklin and R. E. Benn, "A Ten Channel Infrared Spectrograph," *Jour. Optical Soc. Am.*, Vol. 41, No. 2, pp. 76–79 (1951).

Electronic equipment is also used to control the infrared reference sources or cones. These cones were developed to exhibit the properties of a black-body in which temperature equilibrium is assured. The construction of a reference cone consists primarily of a wire helix. The helix is part of a bridge circuit which will balance only when the cone is at a specific temperature. The unbalance voltage of the bridge is amplified and used to control the power supplied to the cone for heating purposes.

The use of potassium bromide prisms, which are extremely hygroscopic, requires that the instrument be sealed and filled with dry nitrogen. Automatic equipment is therefore included to release the excess



Cathode-ray tube display of spectral intensities.

gas as the airplane ascends and replenish it from an auxiliary nitrogen supply during the descent. The dry nitrogen atmosphere, incidentally, is very beneficial to the operation and life of the electronic equipment housed within the instrument. The channel amplifiers containing high-impedance circuits were therefore placed in this location.

This discussion constitutes but a brief resume of the many engineering problems involved in the undertaking which occupied approximately twenty months. It will also be recognized that the successful realization of this complex instrument has only been achieved through the ingenuity of numerous engineers, technicians, and subcontractors. The work was financed by a grant from the Wright Air Development Center of the Air Research and Development Command.

C. W. HARGENS

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

RAPID GAS SAMPLER

A fast acting mechanical device that takes samples of a rapidly changing gas over extremely short intervals—0.2 millisecond or less—has been recently developed by W. J. Levedahl of the National Bureau of Standards. Designed for use in research on the mechanism of engine "knock,"[1] the new gas sampling valve is particularly well adapted to studies of the complex changes that take place in the combustion chamber of an automotive engine.

With the cooperation of the Army Ordnance Department, the Bureau is now studying the reaction kinetics of knocking combustion in an effort to obtain information that will make possible more efficient utilization of automotive fuels. A single-cylinder test engine, in which a wide range of operating conditions may be simulated, has been constructed, and the data thus obtained on the igniting fuel-air mixture are being used to correlate the knocking characteristics of fuels with their chemical structure.

In order to obtain significant data on the chemical processes taking place in the cylinders, some knowledge is required of the proportions of reactants and products present at various times during the combustion cycle. However, multistage combustion reactions of this type are especially difficult to study because of the very short duration of each stage (between 0.1 and 1 millisecond), the extreme complexity of the reaction mechanism, and the fact that many of the important particles are free radicals whose half-lives are probably 1 millisecond or less. Thus, if the combustion gases are to be sampled, the samples must be taken over extremely short intervals during a single cycle of operation, and the reaction must be frozen almost completely upon removal of the sample from the combustion chamber. At the same time, the samples must be large enough for a complete analysis in a mass spectrometer, and the length of the sampling interval must be accurately known to permit correlation between the gas composition and the other measured quantities. The NBS gas sampler was developed to meet these requirements.

Essentially the sampling valve consists of a flanged piston which moves in an evacuated chamber. The valve chamber is connected at its upper end to a vacuum pump and an evacuated sample container.

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* Communicated by the Director.

[1] For further details, see "Instrumentation for Detonation Research," by William J. Levedahl, *Proc. Instr. Soc. Am.* (1952), Paper No. 52-27-3. See also "An Apparatus for Studying Autoignition of Engine Fuels: Results with Normal Heptane and Normal Hexane," by William J. Levedahl and Frank L. Howard, *J. Research NBS*, Vol. 46, p. 301 (1951), RP 2200.

Its lower end first widens abruptly and then narrows to form a connection with the combustion chamber. Initially the valve is closed manually by pressing the piston down so that its lapped flange seats against the edge of the combustion chamber opening. The short length of piston below the flange extends into the opening, completely sealing it off. The valve is held in this position by a trigger device external to the valve chamber. When the trigger is released by a solenoid mechanism, the pressure of the gases in the combustion chamber accelerates the piston upward through the valve chamber so that it is moving with appreciable velocity by the time its lower edge passes out of the combustion chamber opening. At this time the cylinder gases begin passing around the piston head and continue to do so until the upper edge of the head passes into the smaller-diameter upper portion of the valve chamber. The flange then causes the piston head to seat at this point, completely closing the valve. If bouncing occurs, it is not sufficient to cause the valve to reopen.

While the piston is rising, the connection to the vacuum pump is kept closed so that all the combustion gases pass into the sample container. Meanwhile, an extension of the piston shaft progressively cuts off the passage of light from a flashlight bulb to a small 1P42 photoelectric cell. Thus, when the output of the phototube is differentiated, a square wave indicating the time of opening and closing of the valve is obtained on the screen of an oscilloscope used to present pressure changes within the cylinder. Any bouncing or reopening that might occur would also be made visually evident in this way.

In general, the duration of valve opening and the total mass of gas admitted are dependent upon cylinder pressure at the time of the valve opening. Thus, for example, if the pressure is 80 psi., about 1 milliliter of gas (reduced to standard temperature and pressure) will flow in 0.2 millisecond, while at 500 psi. the flow is approximately 4 milliliters in 0.1 millisecond. This decrease in the duration of sampling with increasing pressure is advantageous inasmuch as the reactions studied occur more rapidly at the higher densities.

A "LOUDSPEAKER" CLUTCH

A fast-acting clutch that works on the same moving-coil principle as the electrodynamic loudspeaker of an ordinary radio receiver has been developed by the National Bureau of Standards. The new clutch is activated by applying direct current to a coil located in a constant magnetic field. The force resulting from the interaction of the coil current and the magnetic field moves the coil and causes the clutch output disk to be pressed against the rotating input members. In an experimental model constructed at NBS, full output-shaft torque—10

ounce-inches maximum—was attained in less than a third of a millisecond after application of the actuating voltage. Developed by Jacob Rabinow, head of the NBS Electromechanical Ordnance Division, the NBS "loudspeaker" clutch offers advantages for certain instrumentation and computer applications calling for fast response. Possible uses include rapid starting and stopping of magnetic wire or tape recording media in high-speed electronic computers, like the Bureau's SEAC, and high-speed switching in telephone dial systems.

The loudspeaker clutch is inherently capable of faster response than electrical or magnetic friction clutches of conventional design. This is because, in such conventional clutches, time as well as energy is required to build up a field after the actuating voltage is applied. In the loudspeaker clutch, on the other hand, the necessary magnetic field is already in existence before the clutch is actuated. The inductance of the moving coil in the loudspeaker clutch is normally small, and by means of "bucking" or compensating coils can be made negligible; this means that the time required for the coil current to build up so as to press the clutching surfaces together can be made equally negligible.

In the NBS experimental model, the magnets and coils rotate with the clutch input shaft. Although a permanent magnet could be used, an electromagnet was chosen; the clutch can thus be easily cleaned of any accumulated magnetic particles when the magnetizing current is shut off. Two pairs of slip rings transmit the magnetizing and actuating currents to the rotating assembly. A thin flexible diaphragm is fastened to the actuating coil. When the voltage is applied, the coil jumps forward and presses the clutch output disk between the rotating diaphragm and an also-rotating backing disk. Vent holes in the output disk, diaphragm, and case assist cooling and avoid compression or rarefaction of air by the diaphragm.

Compensating coils are attached to the pole faces of the electromagnet, close to the actuating coils. All the coils are connected in series so that the current is the same in all windings, but the direction of current flow in the compensating coils is opposite to that in the actuating coils. This arrangement cancels out the inductance of the actuating coil to the extent that the coupling between the coils approaches unity, and thus minimizes the time required for current to rise in the coil after the actuating voltage is applied. In the model tested at NBS, the compensating coils reduced the 1-kilocycle inductance of the actuating coils to 0.15 millihenry from an uncompensated value of 2.4 millihenries.

Tests were made to determine clutch response speed and maximum torque, as well as the effectiveness of the compensating coils. For the response-time tests, an ordinary crystal phonograph pickup served as the sensing element, its needle resting in a small depression in the clutch output shaft. An oscilloscope was connected to the actuating switch

and to the crystal pickup so that the lapse of time from the closing of the switch to the beginning of output-shaft motion could be observed. With 300 volts applied both to the electromagnet field coil and to the actuating coil, the output shaft began to move in 200 microseconds and reached full speed in 300 microseconds. With 100 volts on the field and 215 volts on the actuating coil, the corresponding delays were 500 and 625 microseconds.

The actuating voltages required for fastest response are needed only momentarily; to avoid overheating and burnout they must be reduced promptly after the clutch is up to speed. Maximum safe values of current for continuous operation were determined to be 0.2 ampere for the electromagnet field coil and 0.5 ampere for the actuating coil. At these currents, the maximum output torque—the torque at which slippage occurs—was found to be 10 ounce-inches.

Effectiveness of the compensating coils was tested by measuring response times with coil connections reversed, so that the compensating coils aided the actuating coils instead of bucking them. Under the conditions of the test, this increased the delay time from 600 microseconds to 900 microseconds. Indications are that with the regular connection—coils bucking—clutch response time is limited by mechanical rather than electrical factors.

The principle of the NBS loudspeaker clutch may be expected to find application in specialized equipment where light loads must be accelerated at an extremely rapid rate. A similar principle has already been used at the Bureau to advance rapidly frames of photographic film at random intervals of time. Although the actuating mechanism rotates in the experimental clutch model described, an alternative design has been drawn up in which the actuating means are stationary and operate the clutching means through thrust bearings.

CONSTANT-AMPLITUDE OSCILLATOR

A constant-amplitude oscillator, developed by N. C. Hekimian of the National Bureau of Standards, provides an r-f voltage that remains reasonably stable regardless of changes in tube parameters, supply voltage, heater voltage, or load impedance. The device consists essentially of a conventional oscillator with a diode connected across its output terminals. Output stability is provided by a biased control clamper tube sharing the same plate-dropping resistor with the oscillator.

In electronic equipment such as exciters, signal generators, and high-quality communication receivers, the adjustable-frequency oscillators should be stabilized against changes arising from possible variations in circuit components. Automatic volume control systems have been

modified to provide the stabilization, but they have proved ineffective for class-C oscillators because of the self-biasing feature associated with this class of operation. Fixed-level clipper circuits have also been utilized but the output waveform normally contains strong, undesirable harmonic components.

The NBS constant-amplitude oscillator does not suffer from these difficulties to any appreciable degree. The circuit is designed to maintain the apparent grid-plate gain of the oscillator reasonably steady. The input and output capacitances are held practically constant, and the clipping of the output waveform is reduced to very low levels. The moderate amount of clipping that does occur is due only to the diode detector across the oscillator output.

The oscillator developed by NBS was originally designed as a fixed-frequency local oscillator in a gain-stable receiver. It utilizes both sides of a 12AT7 twin-triode as the oscillator and clamper tube, respectively, and a 6AL5 diode detector. The grid tank circuit is composed of a fifth overtone crystal (30 Mc), a frequency-shifting trimmer, and a crystal-peaking coil. A coil in the plate circuit of the oscillator section of the 12AT7 is designed to resonate at the crystal frequency.

The clamper tube is initially biased in the region of plate cut-off. The oscillator output—after rectification by the diode—is applied as a positive voltage to the grid of the control tube—the other half of the 12AT7. When the oscillator output voltage reaches a sufficiently high level, the clamper begins to draw plate current and causes a reduction of plate voltage. Because the oscillator is connected to the same plate-dropping resistor, it too suffers a reduction of plate supply. Thus, the tendency of the clamper, as the oscillator output voltage increases, is to maintain the final output voltage at a relatively fixed level.

The bias for the clamper tube is usually obtained from a low impedance source of constant potential. In the test model of the NBS constant-amplitude oscillator, the bias voltage was obtained from batteries with internal resistances of about 5 ohms. Normally, the bias voltage is adjusted so that the control tube is always slightly conductive at the minimum level of oscillator voltage anticipated. Hence, positive control action is available at all times. Under these conditions, the best regulating characteristics are obtained with the largest plate-dropping resistor that still permits the desired plate voltages. A plate-dropping resistance of 40,000 ohms was found to give satisfactory operating conditions for the NBS stabilized oscillator.

An experimental investigation of the effect of different plate-loading resistances on the oscillator output voltage shows that a resistance of 10,000 ohms permits a 12-volt variation for a change in plate supply from 200 to 350 volts. A plate-load resistance of 40,000 ohms, however, restricts the output variation to 1.5 volts with the same change in

plate supply. In both of these instances the bias supply voltage was held at 18 volts.

An example of the effect the clamper bias voltage has upon the performance of the circuit is illustrated by a graph of output voltage versus supply voltage for several values of bias. A bias of 25 volts will permit the output to vary between 12 and 17 volts for a change in supply from 200 to 350 volts; whereas a bias voltage of 18 volts will result in a change of only 1 volt in the output for the same change in supply. In both of these instances, the plate-loading resistance was 40,000 ohms.

The determination of the constancy of amplitude versus supply voltage reflects to some extent the increased stabilization against tube parameter changes. A graphical determination of operating characteristics will certainly assist in determining the best values for the parameters to help make the circuit perform the function for which it was designed. In most cases, however, it may be advisable to determine circuit values roughly and then adjust the actual oscillator for optimum operation. In practice the output voltage is determined largely by the bias voltage with the plate-loading resistor adjusted for satisfactory regulation. Care should be taken to allow for sufficient dissipation in this resistor—a power-type is generally nesessary.

Improved clamping may be obtained by employing a voltage-multiplier type of rectifier to drive the clamper tube. Thus a greater ratio of d-c control bias to r-f output is obtained and results in better regulation. Also, if a power amplifier replaced the clamper portion of the 12AT7, its greater plate-current capabilities will result in more positive control action. Further, if the clipper diode is fed from the final output of the oscillator-buffer circuits with more gain included between the oscillator and the rectifier-control circuit, the sensitivity to small changes in output is increased and improved stability results.

NOTE: For additional details, see "Constant-Amplitude Oscillator," by Norris C. Hekimian, *Electronics*, p. 164, July (1951).

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING
December 17, 1952

The stated monthly meeting was held on December 17, 1952, in the Lecture Hall of The Institute. S. Wyman Rolph, President, called the meeting to order at 8:20 P.M. Approximately 385 persons attended.

The President stated that the Minutes of the monthly meeting in November are being printed in full in the December JOURNAL and will be submitted for approval next month.

The Secretary reported that according to Article IV, Section 5, of the By-Laws "Nominations for Managers shall be made in writing at the stated meeting in the month of December. Each nomination paper must be signed by at least two members, who shall certify that the candidate will serve if elected. After the nominations are closed, the President shall appoint three members, who are neither officers nor nominees, to act as tellers of election. . . ." Ballots may be cast until 8 P.M. on the day of the election (January 21st). He then read the following nominations already made in accordance with the above provision:

A. Felix duPont, Jr.	Ralph Kelly	Charles Penrose
J. G. R. Heckscher	Lionel F. Levy	R. G. Rincliffe
Clarence L. Jordan		Philip H. Ward, Jr.

The President called for nominations from the floor. There being none, he declared the nominations closed. Dr. George S. Crampton, Dr. Joseph Hepburn, and Mr. Howard Stoertz were appointed tellers.

The Secretary reported the following elections to membership since October 31st:

Active	34
Associate	46
Student	20

and a total Institute membership of 6601 as of November 30, 1952.

He read the names of those who have been members of the Institute for twenty-five consecutive years and who have received Silver Cards of Membership. (These are listed under the Membership Section of the JOURNAL.)

The Secretary announced that the annual James Mapes Dodge Lectures will be given on the afternoons of January 13th and 14th by Dr. I. M. Levitt, Director of the Fels Planetarium of The Franklin Institute, whose subject will be "Space Travel."

Dr. Allen cited the comparative evening attendance at the Christmas Planetarium demonstration for the first week of December. For the current year this attendance was about eight times that of the past three years. This increase is largely due to a collaboration with the Philadelphia Council of Churches. He also noted the Institute's large number of television and radio programs, all of which undoubtedly are helping our Museum and Planetarium attendance. The President mentioned the excellence of the December Planetarium demonstration and suggested that everyone come to see it.

The President then introduced the speaker of the evening, Dr. Nevin Elwell Funk, former Executive Vice-President of the Philadelphia Electric Company. Dr. Funk, who is active on Institute committees, showed some beautiful colored pictures taken when he visited the British Isles. These included some of the important landmarks of Scotland, the Cathedral towns of England, Cambridge and Oxford Universities, and London. Dr. Funk's commentary was most interesting and frequently amusing.

The meeting adjourned at 10:05 P.M. with a rising vote of thanks to Dr. Funk.

HENRY B. ALLEN
Secretary

MEMBERSHIP

NEW GOLD CARD MEMBERS HONORED

On November 7th the Institute honored two men who have been continuous members of the Institute for 50 years, Kern Dodge and John P. B. Sinkler, both of Philadelphia. Mr. Dodge was represented by his son Donald Dodge. Mr. Rolph, the President, presented gold keys and gold cards to the gentlemen at the dinner in Franklin Hall when Dr. G. H. Clamer, Vice President and Chairman of the Executive Committee, and a past gold key holder, reminisced on his 61 years of membership. Other past gold key holders present were Fred H. Colvin, 1888 and J. H. Granbery, 1895.

Mr. S. Wyman Rolph, President of the Institute, presenting gold cards and keys to Mr. Donald Dodge, representing his father, Mr. Kern Dodge, and Mr. John P. B. Sinkler, in recognition of fifty years of continuous membership in The Franklin Institute.

The following brief biographies of the two new Gold Card members will serve to bring some of their accomplishments to the attention of those members who were not able to attend the dinner.

KERN DODGE. Born in Chicago and now a resident of Philadelphia, Kern Dodge received his early education from The Germantown Academy and in 1901 was graduated from The Drexel Institute of Technology; last year he was one of sixty selected from 10,000 graduates to receive a Citation at Drexel's 60th Anniversay celebration.

Soon after being graduated, Mr. Dodge formed a partnership with Charles Day under the firm name of Dodge and Day. Since 1911, when he sold his interest in the business, Mr. Dodge has maintained a private practice as a consulting engineer.

Closely associated with Link-Belt Company for many years, a company founded by his father James Mapes Dodge, Kern Dodge, a director, designed a switchboard still in use today for the Nicetown plant of this company.

A staunch supporter of civic betterment, he was appointed in 1932 Director of Public Safety in Philadelphia, acted as president of the National Council for American Education, and was a director of The National Economic Council.

Mr. Dodge holds life memberships in the following professional and other organizations: American Society of Mechanical Engineers, American Institute of Electrical Engineers, and

The New York Electrical Society. He is also a member of the Board of Trustees of Germantown Academy, The Board of Managers of Moore Institute, a director of the Alumni Association of Drexel Institute, and a Dean of Directors of the Keystone Automobile Club.

JOHN P. B. SINKLER. A native of Philadelphia, John P. B. Sinkler was graduated from The University of Pennsylvania in 1898 with a Bachelor of Science degree in architecture. Supplementing this education, he also studied architecture in Paris. From 1904, Mr. Sinkler practiced architecture with E. Perot Bissel.

Among the buildings designed by the 77 year old architect are The Confederate Memorial Institute, Richmond, Virginia; the original buildings of The Abington Memorial Hospital; the Germantown Municipal Building; the Chestnut Street Pier; Girard Piers, 3 and 5 North; and Funfield Recreation Center. From 1920 to 1924 Mr. Sinkler acted as City Architect of Philadelphia and from 1932 to 1936 he was Director of the Department of City Architecture. In addition, Mr. Sinkler was a member and secretary of the first zoning Board of Adjustment.

Also a member of the Board of the Philadelphia City Institute, he is a Fellow of the American Institute of Architects and a past president of the Philadelphia chapter.

The following are Gold Card members, having belonged continuously for 50 years or more. The dates are years of election to membership.

Arthur W. Howe '84
Alexander Sellers '84
William R. Armstrong '85
William Henry Bower '85
Fred H. Colvin '88
G. H. Clamer '91
Pierre S. duPont '92
Walter T. Lee '92
J. H. Granbery '95
Edward Woolman '95
Alan Wood, III '96
T. C. McBride '97
D. S. Jacobus '99
John J. McVey '99
E. A. Muller '99
C. C. Tutwiler '99
Frederic A. Delano '00
James M. Caird '01
Gano Dunn '01
Kern Dodge '02
John P. B. Sinkler '02

The following are Silver Card members, having belonged continuously for 25 years or more. The dates are years of election to membership.

1903
Charles H. Howson

1904
Frank S. Busser
C. Mahlon Kline

1905
George R. Hall

1906
John L. Cox
Francis J. Stokes

1907
Darthela Clark
Frank Shaw Clark

1908
Herbert Berliner
George W. Borton
W. M. C. Kimber
John B. Rumbough

1909
Theobald F. Clark
Walton Clark, Jr.
J. B. Klumpp
James McGowan, Jr.
George Missimer, Jr.
T. Edward Ross

1910
Richard B. Brown
Joseph S. Hepburn
Joseph W. Lippincott
Horace P. Liversidge
J. D. Shattuck
J. M. Weiss

1911
Seth B. Capp
E. M. Chance
W. H. Fulweiler
Charles Penrose
Frederic Rosengarten

1912
James Barnes
Morris Llewellyn Cooke
Edwin Elliot
Herbert E. Ives
Walter Palmer
Julian S. Simsohn
Howard S. Worrell

1913
Edward Bartow
George L. Coppage
Irenee duPont
Howard N. Eavenson
S. B. Eckert
Ralph E. Flanders
N. E. Funk
J. Kearsley M. Harrison
Lionel F. Levy
E. Mallinckrodt, Jr.
Frank M. Masters
Franklin P. McConnell
William Maul Measey
W. Chattin Wetherill
D. Robert Yarnall

1914
Russell L. Brinton
George S. Crampton
Ernest L. Huff
K. G. Mackenzie
Marshall S. Morgan
J. Edward Patterson
George Wharton Pepper

1915
Raymond Foss Bacon
C. E. Bennett
Thomas D. Cope
John C. Cornelius, Jr.
H. Jermain Creighton
Thomas W. Elkinton
Richard Howson
S. Leonard Kent, Jr.
E. F. Kingsbury
W. Wallace McKaig
Walter F. Rittman

1916
Henry B. Allen
James G. Detwiler
Zay Jeffries
Thorsten Y. Olsen
Frank H. Sauer
B. E. Shackelford
H. B. Spencer
Arthur Synnestvedt
Leo Wallerstein
Charles N. Weyl

1917
A. D. Chambers
Francis J. Chesterman
Walter S. Crowell
P. T. Dashiell
William J. Fitzmaurice, Jr.
Rolfe E. Glover, Jr.
Harold Goodwin, Jr.
Arthur W. Lowe
Frank S. MacGregor
Edward E. Marbaker
Francis F. Milne, Jr.
C. H. Quinn
Sylvan D. Rolle
Robert L. Wood
Henry Woodhouse

1918
Charles H. Colvin
Charles H. Masland, 2nd
Walter O. Snelling

1919
Charles E. Brinley
Henry Colvin, 2nd
E. A. Eckhardt
Edwin D. A. Frank
George W. Furness
Frank H. Griffin
Hiram S. Lukens
H. Conrad Meyer
J. Howard Pew
Edmund G. Robinson
L. K. Sillcox
W. Leigh Smith
William Tiddy
Lucien Yeomans

1920
Robert Cameron Colwell
Howard W. Elkinton
Francis W. Hartzel
Chester Lichtenberg
P. S. Lyon
Haviland H. Platt
Carl D. Pratt
Winthrop R. Wright

1921
Roland L. Andreau
Leonard T. Beale
J. Ed. Brewer
Frederic Palmer, Jr.
Howard Stoertz
James Stokley
Haakon Styri
Edward R. Weidlein
S. Weinberg

1922
C. Douglas Galloway
Joseph F. Greene
L. H. Kinnard
L. W. McKeehan

1923
C. S. Chrisman
William Dubilier
Henry Clay Gibson
John Tracy Lay
Howard S. Levy
William C. Melcher, Jr.
John F. Metten
Lewis F. Moody
Richard H. Ranger
Jessie A. Rodman
Edwin G. Sagebeer
Coleman Sellers, 3rd
H. Birchard Taylor
Francis B. Vogdes
Joseph J. Vogdes

1924
Peter Abrams
H. Carl Albrecht
Clement Starr Brinton
Karl K. Darrow
Lee Davidheiser
William G. Ellis
T. R. Harrison
Valentine Hiergesell
Karl F. Oerlein
Edward B. Patterson
Harold Pender
Nicholas G. Roosevelt
Alfred O. Tate
Walter C. Wagner
James Lloyd Weatherwax
Alexander Wilson, 3rd
William Zimmermann

1925
Jack G. Binswanger
William L. Brown, 3rd
Marion Eppley
John Graham Foley
J. V. Giesler
C. R. Kraus
J. Kenneth W. Macalpine
William Stanton Monroe
W. F. G. Swann

1926
Charles B. Bazzoni
Neils Bohr
O. E. Buckley
William Findlay Downs
Henry C. Evans
Edward L. Forstall
Lewis H. Hendrixson
Charles S. Leopold
Alfred L. Loomis
J. H. Manning
William Clarke Mason
Clement B. Newbold
O. M. Patton
Charles S. Redding
Dexter N. Shaw
Sydney L. Wright
Charles S. Wurts, Jr.

1927
William Adam
T. K. Cleveland
Milton W. Deisley
William DeKrafft
Hiram B. Ely
Leslie Griscom
John W. Harsch
Alfred Iddles
John Van Gasken Postles
George Rosengarten
Philip Sporn
Charles A. Stanwick
Charles A. Young

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, DECEMBER 17, 1952

ACTIVE FAMILY

Howard W Schinlever

ACTIVE

Henry Beck, Jr.
Stanley Bilker
William L. Brehm
John P. Buchanan
George H. Fischer
John P. Gaffigan
Richard B. Goodall
J. A. Ehlinger
Nathan Kleiman
Jan Litynski
S. Earle Moore
Louis J. Orlando
Harry Pein
Leon J. Perelman
Roger F. Samartino
M. W. Smith
F. Palin Spruance
John S. Tanguy
Jacob Trachtman
Lt. Col. John A. Ulrich
Emanuel M. Weinstein
Donald G. Wright

ACTIVE NON-RESIDENT

Frank H. Riddle
Robert W. Shinn
Elmer Stewart

NECROLOGY

John R. Baush '44
Herbert A. Bissout '43
George I. Haggerty '51
John B. Stetson, Jr. '36

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, December 10, 1952.)

HALL OF THE COMMITTEE,
PHILADELPHIA, DECEMBER 10, 1952.

MR. HOWARD STOERTZ in the Chair.

The following report was presented for final action:

No. 3242: Photographs in Glass.

This report recommended the award of a John Price Wetherill Medal each to Robert H. Dalton and S. Donald Stookey, of Corning, New York, "In consideration of their work in leading to and developing a practical and commercially acceptable method for producing photographs in glass and opalescent designs in glass by photosensitive processes."

JOHN FRAZER,
Secretary to Committee.

MUSEUM

"I was the youngest son and the youngest child but two, and was born in Boston, New England," wrote Benjamin Franklin. His birth is recorded as having taken place on January 17, 1706, so that this is the 247th anniversary of the event, and it presents an opportunity to renew acquaintance with the material associated with the eminent man which is in the possession of the Institute Museum.

Franklin was an indefatigible letter writer, and circumstances compelled him to spend much of his time in travel. His numerous journeys were made by coach over rough roads, under conditions not conducive to comfort or good writing. However, Franklin's life was too well organized to permit this travelling time to be entirely wasted, and so he had constructed a small desk which he could support on his knees, and which afforded him a substantial base on which he might note his observations, write his correspondence, or compose reports of his journey. The little desk which must have accompanied him on many of his missions is one of the intimate relics which the Institute has the good fortune to possess.

While writing of these journeys mention should be made of his odometer, which is also a treasured possession. It will be recalled that, while he was the postmaster of the colonies, he marked the Boston Post Road with mile stones. These stones were located by means of this odometer which was attached to the wheel of his coach and measured the distance travelled. The odometer must have been employed to measure off the miles marked by similar stones along the Lancaster, Haverford, Gulph, Ridge, and Bristol roads. At a later date this same odometer was used by Thomas Jefferson.

The other activities of the versatile Franklin are happily recalled by other interesting mementos. His great achievements in electricity are represented by two interesting pieces of apparatus. The first of his many electrical experiments was conducted with a glass tube, rubbed with a silk handkerchief to leave a positive charge on the glass surface. Primitive and inadequate as the apparatus may appear when judged by modern standards, the visitor who looks upon this glass tube should do so with reverent eyes, for results were obtained with it which justified the acquisition of superior facilities. We do not know whether Franklin imported his friction-electric machine, or whether he had it constructed by a local carpenter. Certainly there is nothing in the construction that would overtax the craftsmanship of any carpenter. Its rigid simplicity is such as to provoke astonishment that so much could have been accomplished with its aid. No one who examines the machine can turn away from it without additional respect for the genius who was capable of deducing such theories from the limited results this apparatus provided.

V.2
195

One of the practical applications of Franklin's electrical discoveries was the lightning conductor for the protection of buildings. What is believed to have been one of the first lightning rods erected under Franklin's supervision was found on the home of John Wister on High (now Market) Street. A portion of this rod is on public exhibition.

Franklin was proud of his trade. The famous epitaph he wrote for himself indicates his wish to be remembered as a printer, and no collection of his personal relics would be complete without some of the tools of his trade. The Museum is fortunate enough to have several. Interest is held by the composer's "stick" he used when setting the type for *The Religion of Nature* while working for Palmer, whose famous printing house was situated in Bartholomew Close, London. Franklin learned some of the finer points of his trade in this prosperous establishment and, after he had returned to Philadelphia, we perceive that his knowledge was applied to the tools of his trade. He applied his skill for improving and refining whatever came to hand to these tools. He proudly recounts in the story of his life how he made a new mould and struck matrices in lead with the type in Kreimer's shop where he worked. His crude mould might be compared with others in the Graphic Arts Section.

Of all the books he printed none has preserved Franklin's homespun philosophy as copiously as *Poor Richard's Almanac*. An issue of this annual was selected from the several available because it bears the first mention in print of the recently discovered lightning rod.

Our collection is completed by some household articles that formed part of the more intimate side of life. The only thing which seems to have been a useless decoration in a life that was eminently practical, is the court sword presented to him while he was in France.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A. M. until 5 P. M.; Wednesdays and Thursdays from 2 P. M. until 10 P. M.

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ARSOVE, MAYNARD G.: The Algebraic Theory of Linear Transmission Networks.
MANDEVILLE, C. E.: Slow Neutron Induced Radioactivities of Nuclear Explosions: III.
CHANG, CHIEH-CHIEN, BOA-TEH CHU AND VIVIAN O'BRIEN: Asymptotic Expansion of the Whittaker's Function $W_{k,m}(z)$ for Large Values of k,m,z.
LANGHAAR, HENRY L.: On Torsional-Flexural Buckling of Columns.
BOLIE, VICTOR W.: Powerless Glides Along Zero Curvature Flight Paths.
ELDER, TAIT AND JOHN STRONG: The Infrared Transmission of Atmospheric Windows.

BOOK REVIEWS

CLOUD CHAMBER PHOTOGRAPHS OF THE COSMIC RADIATION, by G. D. Rochester and J. G. Wilson. 128 pages, illustrations, 21 × 28 cm. London, Pergamon Press Ltd.; New York, Academic Press Inc., 1952. Price: L3.10s; $10.80.

To those who have had limited experience in interpreting cosmic ray cloud chamber photographs, and to those who are starting out on such fields of investigation, this book will serve in a most valuable capacity. In his Foreword to the book, Professor P. M. S. Blackett, referring to cloud chamber photographs, writes: "An important step in any investigation using these methods is the interpretation of a photograph, often of a complex photograph, and this involved the ability to recognize quickly many different types of sub-atomic events. To acquire skill in interpretation, a preliminary study must be made of many examples of photographs of the different kinds of known events. Only when all known types of event can be recognized will the higherto unknown be detected." All who read the book will join with Professor Blackett in recognizing that, in facilitating the acquirement of the essential skills for cloud chamber work, namely that of interpretation and that of recognition and attainment of high technical quality, the authors have succeeded brilliantly.

The book is divided into six sections.

Section 1, "Technical Features of Operation," contains some fourteen pages devoted to the illustration of various types of cloud chamber photographs, with emphasis on technical features and such phenomena—distortion at the edge of the piston, turbulent distortion, effects of contamination, etc.—as play important roles in effective cloud chamber operation. The descriptions of the apparatus associated with the photographs are quite detailed and provide a mine of information, particularly for the novice in this realm of experimentation.

Section 2 comprises about 28 pages and is devoted to electron and cascade showers. The interpretation of the photographs is carried out in considerable detail, both as regards the nature of the events and the magnitudes associated with them. Incidentally, this section contains a reproduction of Anderson's classic experiment, from which he inferred the existence of the positive electron.

Section 3 is devoted to slow μ-mesons and the decay of such mesons. Some 15 pages of examples are given, with detailed illustrations, and include some very beautiful photographs showing the death of mesons, both with and without the accompaniment of decay electrons. Again the photographs contain detailed descriptions and quantitative estimates of magnitudes such, for example, as the energies associated with decay electrons.

Section 4 devotes some 32 pages to nuclear disintegration and interaction of secondary particles. Among other things of interest, it contains many photographs taken with large numbers of lead plates in the cloud chamber. In addition to phenomena of nuclear disintegration and allied phenomena, there are pictures of penetrating showers.

Section 5, comprising some 20 pages, is devoted to V-particles and is rich in qualitative and quantitation associated phenomena.

Section 6 comprises a very beautiful picture of a heavy cosmic ray particle passing through four slabs, one comprising 6 mm. of carbon and the others 6 mm. of lead each.

The book is presented in excellent form and the clarity of the pictures is exceptional. The authors have also enhanced the interest of the publication by including photographs of historical interest.

Finally, it is worthwhile calling attention to an aspect of such a group of pictures which, while presumably always implicit in the mind, is apt not to be recognized in its full force. This aspect is recognized in Professor Blackett's Foreword in the following sentence: "If one asks why some of these complicated events happen, one may be led into the subtle intricacies and uncertainties of modern fundamental theoretical physics, but if the experimenter contents himself with asking how they happen, then these pictures, and the attached commentaries, are an ideal guide to the world of energetic elementary particles." W. F. G. SWANN

ANTOINE LAVOISIER, by Douglas McKie. 440 pages, illustrations, 14 × 22 cm. New York, Henry Schuman, Inc., 1952. Price, $5.00.

Lavoisier was the Copernicus of chemistry who reversed the previously assumed loss of phlogiston into a gain of oxygen. All chemical students are aware of this achievement and, too often, knowledge of the distinguished scientist ends abruptly at this point. Those students of chemistry (and of life) who accept this foreshortening of knowledge are the poorer, for Lavoisier's romantic life makes fascinating reading. He combined the gifts of experimenter with those of administrator and financier. In the same month of 1768 in which he was elected to the Academie he was appointed to the "Ferme Générale," that hated commercial organization entrusted by the King with the task of collecting taxes. Always interested in public affairs, he assisted Turgot, the Minister of War, in his efforts to increase the production of saltpeter, thereby becoming the mentor of Eleuthère Irénée Dupont, who was the founder of the great chemical industry in America. Although deeply engrossed with public affairs, Lavoisier devoted one day of his busy week to experiments. During the French Revolution he fought bravely for the security of scientists and the Academie. However, his great contributions to science and the public good were completely overshadowed by his association with the "Fermiers" and he was condemned to the fate which engulfed them all. He was executed in 1794 in his 52nd year.

Berthelot, Lavoisier's biographer, was exceedingly bitter about the silence and inaction of the scientist's friends, like Monge, Morveau, and Fourcroy, who might have done much had they united their efforts to save his life. Dr. McKie is not so free with accusation and condemnation. He is at all times judicial, and tries to do justice to those who feared to raise their voices against the general opinion.

Dr. McKie, who is Reader in the History of Science at the University of London, has already written upon Lavoisier's chemical achievements, but in this book he gives a much fuller portrait of a versatile man. Written in an easy style, it is more truly a biography than an inquiry into technical accomplishments and makes excellent reading for the general reader. Since the subject is in competent hands the scientific aspect is free from the blemishes of overemphasis upon the dramatic. The scientific training of the writer serves as no handicap when he has to deal with the social contexture of the times, for the author is an accomplished historian as well as scientist.

There are too few good biographies of scientists that present a complete picture of the man against the background of his times, but this life of Lavoisier ranks among the most satisfactory.

T. C.

AN INTRODUCTION TO THE CHEMISTRY OF HYDRIDES, by Dallas T. Hurd. 231 pages, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1952. Price, $5.50.

In recent years the field of hydride chemistry has had a rapid growth with regard to both academic and industrial interest. From the academic viewpoint the hydrides represent a group of fascinating compounds whose physical and chemical properties show marked changes with the periodicity and valence of the combining elements. Some of the less common hydrides are beginning to find industrial applications such as sources of hydrogen and very pure metals, rocket fuels, reagents for selective organic reductions, and as catalysts in polymer chemistry. While this list does not contain applications of great industrial importance, hydrides that were merely laboratory curiosities a few years ago are now being produced on a large scale. With the great advances in this field taking place in both industrial and academic laboratories, there has been a great deal of information published in the many scientific journals. Consequently Dr. Hurd has devoted this book to trying to coordinate this large amount of scattered information on hydrides.

The main body of the book is concerned with the chemical bonding, structure, methods of syntheses, properties, and uses of the hydrides. The hydrides are arranged in this book as to the location of the combining elements in the periodic table. After the discussion of each compound, a list of pertinent references is given for the reader who is interested in more detailed information. Since the common hydrides like water, ammonia, the hydrocarbons,

and the hydrogen halides are discussed fully in many other texts, they are given only cursory treatment in this volume. However, they are discussed briefly as to their behavior as hydrides. The final chapter is devoted to the various rules of nomenclature currently in use. The appendices contain valuable information on the toxicology, manipulation, and commercial sources of hydrides. There is also a short section on hydrides prepared from deuterium or heavy hydrogen.

The author accomplishes his purpose of coordinating the latest theories of formation, properties, syntheses, and reactions of the hydrides. The work on relating chemical bonding to structure is particularly well done. The book stimulates interest on the academic level for further fundamental investigations, and should arouse interest in industry to try to find new uses for these fascinating compounds. D. H. RUSSELL

STYRENE, ITS POLYMERS, COPOLYMERS AND DERIVATIVES, by R. H. Boundy and R. F. Boyer. American Chemical Society Monograph No. 115. 1304 pages, diagrams, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1952. Price, $20.00.

This monograph on styrene is an interesting and thorough coverage of the subject. The treatise starts with a history of styrene and covers its development from a sticky, amber impractical mass to a clear sparkling solid. The chapters that follow are a compilation of the tremendous knowledge gained in the last decade and a presentation of the kind of information that is necessary for the making of styrene into an attractive, useful, and marketable plastic product.

A large section of the book pertains to the styrene monomer. Such things as the physical properties, the chemistry, methods of analysis and the handling of the monomer are discussed in separate chapters in great detail.

The theory and practice of styrene polymerization are next presented. Typical subdivisions of this chapter are catalyzed polymerization, inhibition of styrene polymerization and polymerization methods. This is followed by several chapters on the physical properties of polystyrene with special emphasis on the optical and electrical properties. The physical behavior of the styrene derivatives are treated in the same thorough manner.

Of particular interest is the section devoted to a survey of patent literature on styrene copolymers. The review extends to January 1, 1951, and is concerned only with patents that describe the preparation of copolymers of styrene. The information obtained in this review is presented in various tabular forms. In Section I the patent references are listed in chronological order while in Section II they are arranged according to a key number. A third section is an alphabetical listing.

The remainder of the book deals with the fabricating and finishing processes used to make polystyrene into commercially useful articles. Special properties resulting from a particular fabrication process are pointed out and discussed.

The chemist and engineers who work with styrene, its polymers and derivatives will find this book to be a valuable guide and an indispensable reference. M. MATTIA

HIGH-ENERGY PARTICLES, by Bruno Rossi. 569 pages, photographs, drawings, 15 × 23 cm. New York, Prentice-Hall, Inc., 1952. Price, $12.50.

A book by an author with strong qualifications, Rossi's *High-Energy Particles* is certain to fill a key position on the modern physics shelf. This is true partly because of the author's great ability in clarifying a difficult subject and partly because "the problem of elementary particles is today *the* problem of physics." Rossi's long devotion to particle studies before and during his association with the Los Alamos Laboratory as well as his present work at the Massachusetts Institute of Technology contribute to his authority in interpreting this branch of modern physics.

One must agree this is a hard theoretical subject. On the other hand, much can be done experimentally and there is a great deal of room for the work of observers, particularly in measuring and classifying cosmic-ray interactions.

Rossi's book is logically organized and stands as an excellent reference or handbook of

key knowledge on the subject of particle interactions. In contemporary physics elementary particles embrace those whose existence is established through experimental evidence. At this writing these include the Photon, Negaton, Positon, Meson, Proton, Neutron and Neutrino. Presented here is a discussion of the relations existing between their characteristic properties, mass, spin, electric charge, magnetic moment. More elaborate interactions manifested in the collision and scattering of cosmic-ray particles with their close relation to 10^{10} ev cascade showers are studied.

The book contains unusual recordings of nuclear events produced in photographic emulsions. A great deal of emphasis is placed on the use of photographic emulsion in recording the trajectories of ionizing particles. Rossi has not taken space in his book to give a detailed account of these photographic processes except in guarding the reader from thinking they are identical to those used in optical photography. References are given covering matters of thick emulsions and high silver grain density. Mention is also made of the most recent tool, the scintillation counter.

In 1949 Rossi co-authored a book on ionization chambers and counters. It is clear from the present volume that his theoretical understanding is backed by this experience in experimental methods. Throughout the book there is a running account of laboratory arrangements accompanying the theoretical discussion and punctuating it with numerical values.

In conclusion one can venture that in spite of the rapid advances which will undoubtedly be made in the realm of elementary particle theory, Rossi's book is established upon a firm foundation of the vast amount of investigation which went before (Rutherford, Chadwick, *et al.*) and it will not soon become obsolete. C. W. HARGENS

INVESTMENT CASTINGS FOR ENGINEERS, by R. L. Wood and D. Von Ludwig. 477 pages, diagrams, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1952. Price, $10.00.

The word "investment," which has replaced the word "precision" in describing castings manufactured by this process, may be defined as the various materials used to enshroud or surround the cavity formed by a master pattern. The actual pouring operation to fill this cavity may be referred to as an "investment" of the melted casting metal in this covering material. There are anywhere from 25 to 50 separate direct operations, depending on the particular process in use, to create an investment casting.

The combined effort and knowledge of an engineering consultant and a manufacturer of the product have produced an up-to-date and quite comprehensive treatment of the processes used in the metal-working industry for the manufacture of investment castings.

The investment process is by no means a new one, it being a modern development of the ancient "lost wax" process and only one of five modern methods for the casting of fluid metals into "precision" shapes. The first chapter of the book, entitled "History and Description of Investment Casting," is appropriately named and located in the text. It serves as an interesting and informative introduction to all that follows. Succeeding chapters discuss in detail the conspicuous traits, economic limitations and the desirability of using this type of casting throughout industry. The text lists eight major industries—ranging from aircraft through instruments and textiles—using investment castings.

Any statement that this book will serve as an everyday and ready reference text is amply justified by its logical arrangement of information, with profuse illustrations and test-proven metallurgical data. However, with due apology to the authors and the publisher, the level of technical presentation is somewhat above the understanding of a student, depending of course upon how far the student has progressed in his course of study and knowledge of foundry "language." The same consideration would apply to any newcomer into this field, even though he had adequate engineering training and experience of a general nature. A glossary of terms defining their usage in the "language" of the foundry would be an enhancing and a desirable adjunct to this otherwise inclusive publication.

Design engineering considerations, relative to attaining clean, precise and uniform cast dimensions, are stressed in all parts of the book. The possible accomplishments and metal-

lurgical limitations are honestly stated and clearly defined in regard to the tolerances and their control obtainable in present design. New techniques, in two chapters, deal with the "Mercast" process in which the use of frozen mercury is made for master patterns. To date, this process and its mechanics have never been fully presented in printed form. Along with the complete index, this text will fully supply any technologist involved in metal fabrication, with a large fund of information on the rapidly developing methods now in use for the production of investment castings.

S. T. FRY

THE EVOLUTION OF CHEMISTRY, by Eduard Farber. 349 pages, illustrations, 16 × 24 cm. New York, Ronald Press Co., 1952. Price, $6.00.

The chemists' elements, the primary substances that in various numbers and proportions compose the vast multitude of different materials in the world around us, now number over ninety, and the names of many of them have become words in everyday use. But science required many centuries to arrive at a knowledge of what substances are to be regarded as elements, as simple uncompounded bodies. The speed of scientific progress has now grown rapid and its applications almost immediate, but the pace was once more leisurely. The historian of science, in his laudable efforts to provide an adequate literature of the subject, frequently loses perspective of the historical view through his efforts to give the book a popular appeal. The result is to lay emphasis upon the spectacular rather than upon the significant achievements.

Chemists have reason to be grateful to Dr. Farber for having written a well-balanced scholarly work on the history of their science. The book begins with an adequate review of the work performed by those early philosophers who dealt with the chemical transformation of materials when chemistry began to emerge as a science. The second part contains a description of the period when chemical systems began to develop in an altogether different social context. The third part deals with the age of specialization and industrialization in which the accomplishments of the professional scientist began to enrich life.

Dr. Farber has very wisely abandoned the practice of reviewing the achievements of an individual when the chronicle happens to arrive at the date of the individual's entry upon the scene. He follows the more realistic practice of interrupting the chronology to record the development of an idea, how the idea developed to a theory and how practice evolved into experimental methods. The outcome is a solid construction to serve as a basis for interpreting the connections between imagination and necessity, between theory and experiment. The book is well illustrated and fully documented. The quotations, especially from the early texts, are happily chosen. The book can be safely recommended to all readers who are seeking a sound scholarly treatment of the subject. The layman may encounter occasional difficulties, but those who follow the allied sciences, such as medicine and mining, will be rewarded in reading the work.

T. C.

STARS IN THE MAKING, by Cecelia Payne-Gaposchkin. 160 pages, 67 plates, 16 × 24 cm. Cambridge, Harvard University Press, 1952. Price, $4.25.

"The man who presumes to talk of the evolution of the stars must needs be an optimist with a sense of humor. And astronomers are incorrigible optimists." So writes Dr. Cecelia Payne-Gaposchkin, the ranking woman astrophysicist in the world and a lady with a sense of humor in one of the most powerful books which has appeared on the popular astronomical horizon. Its impact is bound to be tremendous for it deals with a contemporary astronomical problem which is both fascinating and intriguing—the evolution of the stars and their systems.

In *Stars in the Making*, Mrs. Gaposchkin, Phillips Astronomer at Harvard University, has traced the life span of the stars and the evolution of the galaxies or star systems. In clear, understandable language she treats the stars from their beginnings in the primordial gas clouds to their final days as dark lifeless bodies floating endlessly through space. Mrs. Gaposchkin, with this book, becomes the most lucid and eloquent spokesman for the traditionally abtruse science of astrophysics. She shows a remarkable ability to generalize without loss of authority or fuzzing of detail to achieve the popular scene.

V.2
195

In a way this book is reminiscent of the Rachel Carson book, *The Sea Around Us*. Miss Carson took the sea and wove a fascinating, informative, story around it. Mrs. Gaposchkin shows the same matchless ability in performing the same task for the skies. And if there was ever an astronomy book which could become a "best-seller"—this is it!

This book has the authority of an advanced text but the writing is so effective—the pictures painted so meticulously—and there is such a remarkable blending of the abstruse and technical with the language of the layman, that it is certain of high place in popular appeal.

As an example of the author's complete comprehension of her subject coupled with a magnificent command of English prose let me quote a paragraph from page 16

> The sun, indeed, holds a mirror to the cosmos. Like all other stars it is a globe of glowing gas, hotter and denser within. Its surface is a seething, surging sea of atoms. Plumes of gas float above it; glowing filaments surge upward; shining fountains cascade downward. Giant tornadoes swirl through the surface. Spicules rise and dissipate like darting flames. Dazzling flares blaze up and vanish. A brilliant chromoscope rings it; and around it gleams the aura of the corona. Powerful magnetic forces play across the surface; atoms and electrons spray into space. As it spins on its axis the equator pulls steadily ahead; and across its face proceeds the slow rhythm of the sunspot cycle, waxing and waning every eleven years. The spectacle is impressive in itself. As a mirror of the cosmos it is stupendous. Other stars are doing the same things, and these stellar habits are the clue to their history.

This expressive, living, graphic language is present throughout the entire book.

With this book Mrs. Gaposchkin is destined to take her place beside those great writers on astronomy for the general reader, Sir Arthur Eddington and Sir James Jeans. Like the English authors she permits the reader to look over the shoulders of the scientist as he resolves the problems in astronomy. Her ability to knit the dry facts of cosmology into a fascinating story insures the wide use and long enjoyment of this book by thousands of people.

I. M. LEVITT

BALLISTICS IN THE SEVENTEENTH CENTURY, by A. R. Hall. 186 pages, plates, diagrams, 15 × 22 cm. New York, Cambridge University Press, 1952. Price, $4.00.

From a study of ballistics in the 17th century, Dr. Hall, University Assistant Lecturer in the History of Science at Cambridge University, concludes that the science of ballistics developed in the natural course of technical evolution, not forcibly advanced due to economic necessity. This is an interesting revelation, since heretofore most historians have regarded ballistics as motivated primarily by practical needs.

Dividing his treatise into the background of Science: Government and Industry, and the gunner and his art, internal ballistics, mathematical ballistics and conclusions, Dr. Hall proceeds to show the extremely crude state of ballistics in the 17th century. He demonstrates the impracticality of the scientists' discoveries—how little use the gunner or artillery man could make of current technical advances, and how the gunner had to improvise in order to use his crude weapon. He reveals that philosophers from Galileo to Newton—embracing the whole of the 17th century—used ballistics to test and "to develop their powers for larger and more important researches" in mathematics and dynamics. Making the book more interesting is the fact that the histories of such sciences as mechanics, dynamics and mathematics during the same century are also traced. Of course, the 17th century is sometime back, but it is surprising to realize the unusually crude state of science in those times.

E. W. HAMMER

ELECTRODYNAMICS, by Arnold Sommerfeld, translated by Edward G. Ramberg. 371 pages, diagrams, 16 × 23 cm. New York, Academic Press, Inc., 1952. Price, $6.80.

Sommerfeld's contributions to the physical sciences were greatly influenced and quickened by the developments that were brought about during the turn of the century. The works of Faraday and Maxwell were making tremendous in-roads into the physical philosophy at the time the author was receiving his formal education. It was during this period that the concept of action at a distance was giving way to the field concepts. As the author points out, the single works which had the profoundest impact on his thinking were those of Hertz—in particu-

lar, a paper which was written in 1888 entitled "Forces of Electrical Oscillations Treated by Maxwell's Theory." Much of the order preserved in this early paper has been carried into the first two parts of this volume; however, Sommerfeld has eliminated the cumbersome coordinate calculations of Hertz by employing the more compact vector algebra. Part three is devoted for the greater part to the author's own original works. It covers the development of the four-dimensional form of electrodynamics as an advanced treatment of Maxwell's theory.

After presenting an introduction to the basic concepts of the electromagnetic field, the author proceeds, in the first part, to establish the fundamental equations of electrostatics and electrodynamics. In this connection, Maxwell's equations are presented both in the integral form and in the differential form, emphasis being placed on the great utility of the differential form. Part two continues to expound the phenomena of the macroscopic world. It is given over to a treatment, by Maxwell's equations, of electrodynamic manifestations. Frequency is not included as a limitation; rather, the aspects of the entire usable spectrum (stationary or steady, quasi-stationary and rapidly varying) is presented. A complete section on wave guides and the Lecher Two Wire Line is included in this second part.

Parts three and four may be considered grouped in the same sense that the first two are; that is, the first part in each group is devoted to establishing the concepts necessary to the following part. The kinematics of the theory of relativity and the electromagnetic theory of the electron are the two main subjects presented in the third quarter of the book. Much detail is expended on these subjects after the author had established the validity of the Maxwell equations in the four dimensional world. The remainder of the book rounds out all of the main concepts presented in the earlier portions of the text. The bulk of the last portion is given to the general theory of relativity.

In this translation of Sommerfeld's third volume, the English speaking peoples' store of knowledge on the subject of theoretical physics has been greatly enhanced. Heretofore this classical work has only been available in German. Language-thought barriers which are too often associated with individual translations have been removed with this translation of the entire subject. Thus, an avenue has been opened whereby free access can be made to the thought-provoking material presented in this notable treatise. T. J. BOWDEN

ELECTROCHEMICAL DATA, by B. E. Conway. 374 pages, tables, 16 × 24 cm. Houston, Elsevier Press, 1952. Price, $8.75.

This book is a collection of numerical data from the field of electrochemistry. Its ten chapters fall roughly into three groups: general constants and physical properties; classical systems; and special topics. The general properties chapters contain material, drawn from the larger field of physics and chemistry, that is also useful to the worker strictly in electrochemistry. Dielectric constants, refractive indices, densities, vapour pressures, etc., of liquids and solutions are listed. Crystal ionic radii of the elements, electrical properties of metals, and many similar sets of properties are included. In the section on classical systems are activity and osmotic coefficients of solutions, transport data for electrolytes and colloids, dissociation constants of acids and bases, electrode potentials, etc. The special topics sections contains chapters on the electrical double layer, the electrochemistry of melts at high temperatures, and the parameters of electrode kinetics.

Such a collection of quantitative data, if up-to-date, complete, and accurate, appears to this reviewer to be very valuable. This book seems, on the basis of a few spot checks, to be accurate, contains very recent data, and is as complete as can be expected. Furthermore, it is thoroughly referenced, and can be used as an excellent bibliography in this field. In all respects, it is a book worth having at your elbow if your work is in electrochemistry.

A. D. FRANKLIN

REVIEW OF ELECTRONIC DIGITAL COMPUTERS. Papers and discussions presented at the Joint AIEE-IRE Computer Conference, Philadelphia, 1951. 114 pages, illustrations, 22 × 28 cm. New York, American Institute of Electrical Engineers, 1952. Price, $3.50.

In December 1951, the American Institute of Electrical Engineers and the Institute of Radio Engineers sponsored jointly the first computer conference. These meetings were de-

voted to reviewing the progress in the comparatively new field of large scale digital computers. The publication now reviewed is a collection of the papers presented at the technical sessions of that conference.

While numerous papers have appeared previously in the technical press on subjects pertaining to digital computers, this collection of papers describes, for perhaps the first time, the design features of existing, large scale, digital computers which are in productive operation. Consequently, the subject matter is realistic and definitive, representing the state of the art to the date of the conference.

In terms of their popularly accepted code names, the following computers are described: Univac, Burroughs Laboratory, IBM-CPC, Ordvac, ERA 101, Mark III, University of Manchester, Whirlwind I, EDSAC, SEAC, and BTL Relay. For the most part, each of the contributors describes the over-all design features of his computer, with varied amounts of detail with respect to computing elements, memory systems, and input-output equipment.

Upon reading the book, one realizes that the electronic computing art is on the threshold of some untold development in the future. The tremendous impetus to develop new computers has resulted in purely experimental or laboratory models being called production models and made available for sale. This would most certainly not be condoned by engineers with regard to other items of engineering interest. The developments, however, can lead to a better appreciation of the engineering and design problems which will be encountered in future machines. Of importance in this category is the establishment of standards of performance, especially for computers of basically different design capabilities. Our engineering ingenuity, however, will meet these problems in due time.

S. CHARP

THEORY OF MATRICES, by Sam Perlis. 237 pages, 15 × 22 cm. Cambridge, Addison-Wesley Press, Inc., 1952. Price, $5.50.

This introductory text has been planned for the use of students taking a first course in the theory of matrices. The central pattern of the book is the development of the well known canonical forms. The basic concepts and theorems of vector spaces are given in an early chapter. Equivalence, rank and inverses are treated in detail. Congruence is introduced in connection with the problem of reducing quadratic forms.

Matrices with polynomial elements and matric polynomials are discussed, preparatory to the study of similarity. The Cayly-Hamilton theorem is proved, as well as some of its consequences.

The author's treatment of eigenvalues and their use in determining diagonalization properties, orthogonal similarity, and unitary similarity forms one of the best chapters of the book. The iteration method of finding the roots of the frequency equation of a purely oscillatory dynamical system provides an interesting application.

A concluding chapter presents briefly the algebra of linear transformations on n-dimensional vector spaces in matrix formulation.

In the preface the author expresses the aim of communicating a knowledge of matrix theory to students of engineering, psychology, chemistry and other applied sciences. It is doubtful if his approach achieves this aim. Factorization of rectangular matrices, useful to the psychology student, is omitted. Sylvester's theorem, useful to the student of differential equations, is also omitted. In the problem of reduction of quadratic forms, the motive is almost obscured by the detail.

The author's emphasis on matrices as elements of a field will have great appeal to the student of algebra. Many significant theorems and properties of matrices appear in the exercises. For these reasons, and the author's thorough treatment of algebraic questions, the text is suitable for mathematics majors. It must be admitted that the subject of matrices, extending as it does into linear algebra, transformation theory, and other branches of mathematics, is difficult to cover in such a limited volume.

HOWARD H. BROWN

GLYCOLS, edited by George O. Curme, Jr. and Franklin Johnston. 389 pages, diagrams, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1952. Price, $12.00.

This member of the Monograph series presents an exhaustive coverage of the most recent technology of industrially important glycols. Although these compounds have been known since 1859, it has only been in the last 25 years that they have taken on great economic significance. The principal use of the glycols has been in anti-freezes and explosives, but they are becoming increasingly important in the manufacture of such things as textiles, surface coatings, cosmetics, and drugs. Together with the development of new industrial uses for glycols there have been many advances in the technology of the field. Hence existing books which give the glycols only superficial treatment have become inadequate. Obviously there is a need for an up-to-date comprehensive survey of the field, and this book was written to serve the purpose.

The major emphasis in this book is on the production methods, physical properties, and commercial applications of ethylene and propylene glycol. Their manufacture through the chlorohydrins and by direct oxidation of the hydrocarbons is very thoroughly covered. The book also provides an excellent and one of the only over-all surveys of polymeric derivatives of these two glycols. The rest of the book is given over to the higher glycols, toxicology, chemical analysis, and test methods.

The readability of this book deserves particular mention. The abundant use of tables, chemical formulae, and self-explanatory graphs facilitates reading and shortens the book considerably by omitting much descriptive text. Each chapter is prepared by an expert in the particular field and is thoroughly annotated, with many references to both the classical work and the latest advances in the field. The editors accomplish their purpose of presenting a brief but comprehensive account of the current status of the glycol industry. Although some phases are given only cursory treatment, this treatise is still the most complete and up-to-date compendium of glycol technology. No one actively connected with either production or applications of glycols can afford to be without this valuable book. D. H. RUSSELL

STORAGE TUBES AND THEIR BASIC PRINCIPLES, by M. Knoll and B. Kazan. 143 pages, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1952. Price, $3.00.

Storage Tubes explains the basic theory and operation of the different types of charge-controlled storage tubes used in television, computer and signal-converter applications. Although a number of articles have appeared on the broad subject of electronic storage, this is the first work devoted exclusively to storage tubes.

Although the book is put together in an orderly way, its highly specialized subject matter makes it difficult reading even for engineers. A good feature is the standard method for presenting each tube under these headings: Description, Writing, Reading, Erasing, Polarity and Half-tones, and Additional Considerations. A schematic diagram for each tube does much to clarify the text.

After introductory chapters on basic theory and methods of writing and reading, the four remaining chapters cover in detail the four special types—signal-converter, viewing, digital computer and television-camera storage tubes. Included are the graphechon, radechon, Donal and Langmuir's light valve tube, Williams' tube, Rajchman's tube, the iconoscope, orthicon, vidicon and other similar tubes.

An extensive bibliography and an index add to the value of the text, whose reasonable price is possible through use of the offset process, with unjustified right-hand margins.

E. A. MECHLER

BOOK NOTES

PRINCIPLES OF RADAR, by J. Francis Reintjes and Godfrey T. Coate. Third edition, 985 pages, diagrams, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1952. Price, $7.75.

Although this is the third edition of this work in ten years, the field it covers has changed so rapidly that the present edition has been completely reorganized and nearly all the material has been rewritten. About half the illustrations are new and the book is about twenty per cent longer than the second edition. A new chapter on Radio-frequency Transmitting and Receiving Systems has been added. Dealing with the fundamental concepts and techniques of pulse radar, the book presents the engineering principles of the pulse circuits and the high frequency devices common to nearly all radar systems. It should also be pointed out that the format is new, being printed in double-column letterpress, rather than offset.

A "FIRST BOOK" ON FIRE SAFETY IN THE ATOMIC AGE, by Horatio Bond. 74 pages, diagrams, 14 × 20 cm. Boston, National Fire Protection Association, 1952. Price, $3.00.

Setting down for the first time a program of A-bomb fire safety, this small book should be read by all Civil Defense workers, firemen, fire-protection engineers, and just ordinary citizens who may be the victims of an A-bomb attack. The author has made a comprehensive study of wartime bomb-set fires and his findings are gathered together under such topics as Primary Fire Effects, Secondary Fire Effects, Fire Fighting, Outdoor Storage, Fire Resistive Buildings, etc. A comprehensive bibliography adds to the value of the book, as do the illustrations.

ELECTRICAL FUNDAMENTALS OF COMMUNICATION, by Arthur L. Albert. Second edition, 531 pages, diagrams, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., 1952. Price, $7.00.

Retaining the same scope and basic nature of the original 1942 edition, this second edition of a text for beginning students in electrical communication places emphasis on radio (and television) fundamentals. Definitions and terminology have been changed to agree with the latest standards of the IRE and the AIEE. In its new form, the text should continue to be popular with students and teachers.

TEXTBOOK OF QUANTITATIVE INORGANIC ANALYSIS, by I. M. Kolthoff and E. B. Sandell. Third edition, 759 pages, diagrams, tables, 16 × 24 cm. New York, The Macmillan Co., 1952. Price, $6.50.

The third edition of this standard text has been modernized and expanded. Sections receiving special attention in this revision are those on physical and physico-chemical analytical methods, quantitative separations, organic reagents, electrolytic methods, extraction methods, and separations based on ion exchange and chromatography. A new section on the Brönsted theory of acids and bases has been added to Chapter IV. Although instrumental methods of quantitative analysis have developed tremendously since 1936, when the first edition appeared, the authors believe that the fundamentals of classical analytical chemistry must be learned. They have, therefore, prepared this third edition along the same lines as the previous ones.

FUNDAMENTALS OF ENGINEERING ELECTRONICS, by William G. Dow. Second edition, 627 pages, diagrams, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1952. Price, $8.50.

Students and professors of electrical engineering courses will welcome the second edition of this well-known text on electronic circuitry. The major changes in the present edition are: (1) use of the mks system of units; (2) modernization of the treatment on space-charge control tubes; (3) new principles employed in the ultrahigh and microwave frequency ranges; (4) analysis of semiconductors with reference to the Fermi distribution and the Fermi level. To make room for these changes without increasing the size of the text, some material has been omitted, but most of this omitted material will be incorporated into two other texts soon to be published.

AERIAL MAPPING AND PHOTOGRAMMETRY, by Lyle G. Trorey. Second edition, 180 pages, diagrams, plates, 16 × 24 cm. New York, Cambridge University Press, 1952. Price, $6.00.

With the first edition only two years old, the author is bringing out a second edition primarily to correct errors and misprints. However, a new Appendix (III) gives details of the Kelsh plotter, and the latest available knowledge has been included in the form of notes.

WOOD CHEMISTRY, Vol. 1, edited by Louis E. Wise and Edwin C. Jahn. American Chemical Society Monograph No. 97. Second edition, 720 pages, diagrams, 16 × 24 cm. New York, Reinhold Publishing Corp., 1952. Price, $15.00.

In the second edition, this monograph has been expanded and divided into two volumes. Volume 1 deals with the growth, anatomy and physical properties of wood, the components and chemistry of the cell wall, and the extraneous components of wood. This comprehensive volume is now up-to-date, and should prove valuable to all workers and students in such fields as wood technology, pulp and paper, rayon, cellulose, lacquers and plastics.

METHODS OF STATISTICAL ANALYSIS, by Cyril H. Goulden. Second edition, 467 pages, tables, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1952. Price, $7.50.

Important advances in the field of statistics since 1939—the date of the first edition—have made it necessary to bring out a second, completely revised edition. The book aims to give the student a basic knowledge of statistical techniques, so that he may understand new advances. This aim is accomplished by following a uniform method of presentation, in which a general statement is followed by the algebraic development, and then by a worked-out example. Emphasis is placed on experimental design, although the book covers the general field of applied statistics.

NOTE: The JOURNAL editors wish to call attention to two more titles in the series of reprinted science books sponsored by Dover Publications. These are unabridged, reprinted editions of important scientific works. They are offered at low prices, making it possible for students to own copies at a fraction of the cost of current science books.

SCIENCE AND METHOD, by Henri Poincare. 288 pages, 13 × 20 cm. New York, Dover Publications, Inc., 1952. Price, $1.25 (paper), $2.50 (cloth).

SCIENCE AND HYPOTHESIS, by Henri Poincare. 244 pages, 13 × 20 cm. New York, Dover Publications, Inc., 1952. Price, $1.25 (paper), $2.50 (cloth).

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

VARIATION IN EXTRACTABILITY OF PHOSPHATASE AND NITROGEN IN LIVER OF RATS ON PROTEIN-VARIED DIETS

BY

M. H. ROSS, J. O. ELY, AND JUSTINE ARCHER

The amount of enzyme extracted from a tissue has heretofore been considered to be directly proportional to the amount of enzyme that was present in that tissue *in vivo*. Investigation of alkaline and acid phosphatase activity of liver of rats on protein-varied diets revealed that the amount of alkaline phosphatase removed by aqueous extraction is not necessarily a true measure of the alkaline phosphatase activity in the homogenate.

Male Wistar strain rats of 160–170 g. were put on one of four protein-varied diets (Table I) for 30 days. For the homogenates,

TABLE I.—*Variation in Extractability of Nitrogen in Rat Liver with Protein-Varied Diets.*

Diet of rats	No. of rats	Total nitrogen, mg./100 mg. liver	Nitrogen in aqueous extract, mg./100 mg. liver	Nitrogen extracted, %
Control* (26% protein)	26	3.34† ± 0.028	2.86 ± 0.340	85.5 ± 0.64
18% Protein‡	17	3.15 ± 0.047	2.56 ± 0.706	81.0 ± 1.55
Protein-free‡	16	2.54 ± 0.068	1.99 ± 0.368	80.7 ± 1.58
18% Protein‡ + 0.06% dimethyl-aminoazobenzene (butter yellow)	17	2.95 ± 0.034	2.47 ± 0.335	83.8 ± 0.87

* Fox Food Blox, an Allied Mills product.
† Mean ± standard error of the mean.
‡ Casein (18%, 0%), dextrose (71%, 89%), corn oil (5%), salt mixture (4%), rice bran extract (Vitab) (2%), riboflavin (0.002%).

weighed pieces of liver approximately 1 g. from each of 5 rats maintained on the same diet were homogenized together in the Waring Blender for exactly 1 min. in 100 ml. of distilled water. The homogenates were stored at 4° C. for 3 hr. For the aqueous extracts, a piece of liver of known weight, approximately 200–300 mg., from each rat was ground individually in a glass tube with clean sand and 0.1 ml. of distilled water. 4.9 ml. of water were then added to the finely ground liver paste. This mixture was allowed to stand for 30 min. at 40° C., then centrifuged, and the extract stored for 3 hr. at 4° C.

Total nitrogen and water-extractable nitrogen determined by the method of Ma and Zuazaga (1) showed lower values for livers of rats

fed the three protein-varied diets than for those of rats fed the control diet, as may be seen in Table I. Most of the nitrogen was extracted from every liver, but no direct correlation could be established between the amount of nitrogen extracted and the total amount in the liver before extraction.

Alkaline phosphatase activity (Table II) when calculated on the

TABLE II.—*Variations in Phosphatase Activity* Values in Homogenates, Extracts and Residues of Livers of Rats on Protein-Varied Diets.*

Diet		No. of rats	Alkaline phosphatase activity		Acid phosphatase activity	
			γP/mg. liver	γP/mg. N	γP/mg. liver	γP/mg. N
Control (26% protein)	Total homogenate	35	1.08† ± 0.02	32.37 ± 0.70	14.57 ± 0.12	434.05 ± 3.93
	Aqueous extract	19	0.66 ± 0.03	22.79 ± 1.08	11.25 ± 0.19	394.67 ± 5.62
	Residue		0.42	88.42	3.32	689.9
18% Protein	Total homogenate	35	0.52 ± 0.02	16.61 ± 0.75	14.54 ± 0.50	461.30 ± 15.75
	Aqueous extract	10	0.42 ± 0.03	16.31 ± 0.82	9.88 ± 0.47	386.54 ± 14.70
	Residue		0.10	16.81	4.66	783.2
Protein-free	Total homogenate	35	2.22 ± 0.25	87.56 ± 10.05	14.63 ± 0.45	575.42 ± 17.64
	Aqueous extract	9	0.86 ± 0.08	42.85 ± 2.93	10.14 ± 0.48	512.29 ± 18.94
	Residue		1.36	245.05	4.49	809.0
18% Protein + Butter yellow	Total homogenate	35	0.67 ± 0.04	22.75 ± 1.39	6.20 ± 0.22	210.58 ± 7.65
	Aqueous extract	10	0.40 ± 0.02	16.26 ± 0.82	4.59 ± 0.23	185.46 ± 8.11
	Residue		0.27	56.37	1.61	336.1

* Enzyme activity, expressed as the amount of phosphate liberated by the enzyme in homogenate and in extract was determined directly and in the residue was calculated. Substrate sodium β glycerophosphate, final concentration 5 mg./ml., pH 9.1 and 5.0, temperature 37° C., time 1 hr.
† Mean ± standard error of the mean.

basis of nitrogen content was greater in the residue than in extract or total homogenate. The proportion of alkaline phosphatase extracted, calculated either on the basis of weight of liver or on the nitrogen content, varied with the diet. Therefore, the alkaline phosphatase activity in aqueous extracts can not be regarded as being proportional to the total alkaline phosphatase activity. The total amount of this enzyme in rat liver and also the proportion of loosely bound to more tightly bound enzyme is different under different dietary regimes.

Acid phosphatase activity (Table II) when calculated on the basis of nitrogen, was also greater in homogenates than in aqueous extracts. The activity when calculated on the basis of liver weight was less in the residue than that in the extract. The acid phosphatase activity in livers of rats whose diet contained butter yellow was the lowest, and differed to the same extent from the activity in livers of rats maintained on control diet in homogenate, extract, or residue. Acid phosphatase activity in aqueous extracts was proportional to the total acid phos

phatase activity, whereas alkaline phosphatase activity was not proportional.

Nitrogen content of tissue preparations also has been used heretofore as the basis for calculating enzyme content of the tissue (2). Because of variation in extractability of tissue nitrogen (Table I), the nitrogen content in crude tissue extracts can not be used as the denominator for calculating enzymatic activity (Table II).

The results of this investigation indicate that total homogenate instead of aqueous extract should be used for accurate determination of alkaline phosphatase activity.

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CURRENT TOPICS

Refrigerant Cleans Photographic Film.—The same type of chemical that freezes ice in a refrigerating system and serves as the cooling agent in air conditioning is now proving itself as a better, faster photographic film cleaner.

It's Du Pont "Freon-113" fluorinated hydrocarbon, a nonflammable liquid that eliminates the bothersome, sometimes dangerous, odors of other solvent cleaners. Tests reported by the Motion Picture Research Council, Inc., show "Freon-113" to be "much less toxic, better in some cleaning operations, and equivalent in all other operations tested" in motion picture studio work

Because it's fast-drying, "Freon-113" speeds up film cleaning—as much as 10 to 20 per cent in machine operations. Adaptable to both machine and hand use, it effectively dissolves and washes away gums and oils that "dirty up" photographic film, but does not affect the emulsion of either black and white or color film.

Color motion picture film positives have been cleaned repeatedly with "Freon-113" with no measurable deterioration of color intensity. In the past, effective life of many types of color film has been limited by color fading and running when they were cleaned with other, less selective, solvents.

The versatile chemical, currently available only in 100- and 200-lb. drums, also can be combined with beeswax or cetyl alcohol to make a lubricant for treatment of motion picture film before projection.

A solution of 0.1 per cent of beeswax or cetyl alcohol in "Freon-113" fluorinated hydrocarbon makes a good lubricant. If a higher wax concentration is desired, a co-solvent such as cyclohexane may be added in the ratio of 6.7 per cent cyclohexane to 93.3 per cent "Freon-113" by weight. This combination will dissolve 0.2 per cent of its weight of beeswax.

Some laboratories have found that the addition of a small amount of wax to their "Freon-113" cleaning solution permits cleaning and lubrication of film in one operation.

Although of very low toxicity, "Freon-113" should be used only in well-ventilated work rooms, especially in hand cleaning where the local vapor concentration may be high. Like most solvents, it has a "drying out" effect on the hands, and rubber or neoprene gloves should be worn when the chemical is being used over any extended period.

New Type High-Slip Motor.—A new totally enclosed, fan-cooled, high-slip induction motor, designed for use in acceleration of high-inertia loads such as punch presses, centrifuges, hoists, etc., has been announced by the General Electric Company's Small and Medium Motor Department.

Designated as Type KRX, the new motor is as much as 30 per cent smaller and 40 per cent lighter than conventional totally-enclosed fan-cooled high-slip motors. The space and weight reduction, according to G-E engineers, is the result of the motor's new extended-bar design which provides efficient dissipation of the increased heat normally generated by high-slip motors.

In the new motor, low-resistance rotor bars are extended on one end and

pass through a rotating baffle plate, the engineers explained. Beyond the baffle plate, the bars are brazed to strips of high-resistance metal, shaped to form a radial-blade fan. These fan blades, themselves, provide the higher rotor resistance necessary for high-slip characteristics. Thus, most of the rotor heat develops where there is a direct transfer to the cooling air, that is, outside the motor enclosure.

The engineers pointed out that the reduced heating within the motor enclosure allows a normal air gap between rotor and stator to be maintained. This reduces operating costs, they said, because there is no loss of electrical efficiency.

The Type KRX motor is available in 30 to 150 hp at 900 and 1200 rpm, 5–8 and 8–13 per cent slip. Voltage ratings are 220, 440 and 550.

New Constant-Frequency Control.—A new constant-frequency control designed to hold alternator frequency within 0.001 per cent on motor-generator sets supplying up to 10 kw of power has been announced by the Special Products Section of the General Electric Company.

Developed by the G-E General Engineering Laboratory, the equipment is for industrial and laboratory applications where close frequency control is necessary. It is available in 50, 60, and 400 cycles per second standard in single or three phase, and non-standard frequencies are available on special order.

Specific applications include the prevention of undesired hysteresis and eddy current effects in testing due to frequency variation; a standby source of emergency power for automatic and synchronous equipment; power supply for high-accuracy marine clocks and recorders; and as a plant frequency standard where one 60/400 cycle source supplies power to many parts of a plant and eliminates the need for a number of smaller sources. The control can also be adapted for use with a constant-speed mechanical drive.

The electronic control consists basically of a tuning fork and a phase comparator enclosed in a case approximately 6×24×36 in. It operates on 115-volt, 50/60 cycle power, using approximately 150 watts. The motor-generator requires 115/230 volt d-c. supply. Other ratings are available on special order.

In operation, the output of the alternator of the motor-generator set is held in synchronism with the tuning fork frequency standard by comparing the alternator phase with that of the tuning fork. As the alternator phase begins to lag, the current in the control winding on the main shunt field of the d-c. motor is increased. The control field bucks the main field and increasing control field current tends to cause an increase in motor speed. Thus, the alternator is held in synchronism with the tuning fork signal and does not gain or lose a cycle relative to standard.

Changes in load will cause the output to shift slightly in phase. However, the magnitude of this total shift is always less than plus-or-minus 90 degrees from a set point. An anti-hunt circuit is used to improve stability.

All-Wood Truck Body.—The nation's lumber industry has announced the development of a new all-wood truck body, designed to conserve tight steel supplies and compete with steel truck bodies. The truck body, developed at

the Washington laboratory of the Timber Engineering Company, research arm of the National Lumber Manufacturers Association, is intended primarily for military use. But NLMA President John B. Veach said it also is adaptable to civilian trucks.

An outstanding feature of the new truck body is its exceptional strength, achieved by gluing and bending much of the wood involved. Thin strips of black gum are glued in layers and then bent to form U-shaped framing members which run the width of the body. This laminating process gives the members much greater strength than ordinary timbers. The same principle is used in building wooden ship frames. The truck body has solid side and floor panels which can be mass produced by most woodworking plants.

Three all-wood truck bodies will be made available for tests at the Army's Aberdeen, Md., proving grounds early this year. Every part of the new body has been specially treated with preservatives to make decay virtually impossible. Waterproof glues will prevent the laminated wood from separating under moist tropical conditions. The truck body also is treated to resist warping.

Automatic Hot Coffee Server.—A new automatic hot coffee server, designed to meet the growing needs of business and consumers, has been announced by Rudd-Melikian, Inc., Philadelphia, pioneers of the automatic coffee merchandising industry.

No bigger than an office-type water cooler, the new unit, called the "Coffee Cub," stands ready to serve 100 individual cups of hot coffee in sanitary paper cups without refilling. The ingredients used to make the individually brewed cups of delicious coffee are: Kwik Kafe, Rudd-Melikian's own liquid pure coffee concentrate that is frozen-fresh; liquid pure cane sugar; fresh cream; and hot water.

The firm is introducing its new machine after months of test-marketing the product in key areas across the country, and after years of laboratory and consumer research. The Cub is only 45 in. high, 16 in. wide, and 21 in. deep. It is attractive in coffee and cream colors, and weighs 115 lb. It is made of high quality steel, while all parts that touch the food products are made of stainless steel.

Coffee Cubs are unique, containing both cold and heat generating elements. Refrigeration gives constant protection for the coffee concentrate, cream, and sugar. The heating unit gives a constant supply of hot water used to reconstitute the coffee concentrate.

A coin in the slot, or a flip of a switch, is all a consumer does to get a cup of coffee. After making the selection of the kind of coffee desired, the customer drops a coin in the slot. Down pops a paper cup into a filling well in the front of the machine, and out pours the coffee as ordered. This whole simple operation takes only 5 seconds.